CHAPTER 1

Brainstorming 1

In pairs

Aim: To identify the relationship between multiplication of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Exhibit three examples in the mathematics corner for other groups to give feedback.

Multiplication of numbers in index form	Repeated multiplication
(a) $2^3 \times 2^4$	$\overbrace{(2 \times 2 \times 2)}^{3 \text{ factors}} \times \overbrace{(2 \times 2 \times 2 \times 2)}^{4 \text{ factors}} = \overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}^{7 \text{ factors (overall)}} = 2^7$ $2^3 \times 2^4 = 2^{\square}$ ($7 = 3 + 4$) $2^3 \times 2^4 = 2^{3+4}$
(b) $3^2 \times 3^3$	$\overbrace{(3 \times 3)}^{2 \text{ factors}} \times \overbrace{(3 \times 3 \times 3)}^{3 \text{ factors}} = \overbrace{3 \times 3 \times 3 \times 3 \times 3}^{5 \text{ factors (overall)}} = 3^5$ $3^2 \times 3^3 = 3^{\square}$ $3^2 \times 3^3 = 3^{\square}$
(c) $5^4 \times 5^2$	$\overbrace{(5 \times 5 \times 5 \times 5)}^{4 \text{ factors}} \times \overbrace{(5 \times 5)}^{2 \text{ factors}} = \overbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}^{6 \text{ factors (overall)}} = 5^6$ $5^4 \times 5^2 = 5^{\square}$ $5^4 \times 5^2 = 5^{\square}$

Discussion:

What is your conclusion regarding the relationship between multiplication of numbers in index form and repeated multiplication?

From Brainstorming 1, it is found that:

$$2^3 \times 2^4 = 2^{3+4}$$
$$3^2 \times 3^3 = 3^{2+3}$$
$$5^4 \times 5^2 = 5^{4+2}$$

In general, $a^m \times a^n = a^{m+n}$

Brainstorming 2

In pairs

Aim: To identify the relationship between division of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your findings.

Division of numbers in index form	Repeated multiplication
(a) $4^5 \div 4^2$	$\frac{4^5}{4^2} = \frac{\overbrace{4 \times 4 \times 4 \times 4 \times 4}^{5 \text{ factors}}}{\underbrace{4 \times 4}_{2 \text{ factors}}} = \underbrace{4 \times 4 \times 4}_{3 \text{ factors (Remainder)}} = 4^3$ $4^5 \div 4^2 = 4^{\square}$ ($\square = 5 - 2$) $4^5 \div 4^2 = 4^{5-2}$
(b) $2^6 \div 2^2$	$\frac{2^6}{2^2} = \frac{\overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}^{6 \text{ factors}}}{\underbrace{2 \times 2}_{2 \text{ factors}}} = \underbrace{2 \times 2 \times 2 \times 2}_{4 \text{ factors (Remainder)}} = 2^4$ $2^6 \div 2^2 = 2^{\square}$ $2^6 \div 2^2 = 2^{\square}$
(c) $(-3)^5 \div (-3)^3$	$\frac{(-3)^5}{(-3)^3} = \frac{\overbrace{(-3) \times (-3) \times (-3) \times (-3) \times (-3)}^{5 \text{ factors}}}{\underbrace{(-3) \times (-3) \times (-3)}_{3 \text{ factors}}} = \underbrace{(-3) \times (-3)}_{2 \text{ factors (Remainder)}} = (-3)^2$ $(-3)^5 \div (-3)^3 = (-3)^{\square}$ $(-3)^5 \div (-3)^3 = (-3)^{\square}$

Discussion:

What is the relationship between division of numbers in index form and repeated multiplication?

From Brainstorming 2, it is found that:

$4^5 \div 4^2 = 4^{5-2}$
$2^6 \div 2^2 = 2^{6-2}$
$(-3)^5 \div (-3)^3 = (-3)^{5-3}$

In general, $a^m \div a^n = a^{m-n}$

Brainstorming 3

In pairs

Aim: To identify the relationship between a number in index form raised to a power and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your finding.

Index form raised to a power	Repeated multiplication in index form	Conclusion
(a) $(3^2)^4$	$\overbrace{3^2 \times 3^2 \times 3^2 \times 3^2}^{4 \text{ factors}}$ $= 3^{\underbrace{2+2+2+2}_{4 \text{ times}}}$ (2 is added 4 times) $= 3^{2(4)}$	$(3^2)^4 = 3^{2(4)}$ $= 3^8$
(b) $(5^4)^3$	$\overbrace{5^4 \times 5^4 \times 5^4}^{3 \text{ factors}}$ $= 5^{\underbrace{4+4+4}_{3 \text{ times}}}$ (4 is added 3 times) $= 5^{4(3)}$	$(5^4)^3 = 5^{\square}$ $= 5^{\square}$
(c) $(4^3)^6$	$\overbrace{4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3}^{6 \text{ factors}}$ $= 4^{\underbrace{3+3+3+3+3+3}_{6 \text{ times}}}$ (3 is added 6 times) $= 4^{3(6)}$	$(4^3)^6 = 4^{\square}$ $= 4^{\square}$

Discussion:

What is your conclusion regarding the index form raised to a power and repeated multiplication in index form?

The conclusion in Brainstorming 3 can be checked using the following method.

Example (a)

$$(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$$
$$= 3^{2+2+2+2}$$
$$= 3^8$$
$$3^{2(4)} = 3^{2 \times 4}$$
$$= 3^8$$

Example (b)

$$(5^4)^3 = 5^4 \times 5^4 \times 5^4$$
$$= 5^{4+4+4}$$
$$= 5^{12}$$
$$5^{4(3)} = 5^{4 \times 3}$$
$$= 5^{12}$$

Example (c)

$$(4^3)^6 = 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$$
$$= 4^{3+3+3+3+3+3}$$
$$= 4^{18}$$
$$4^{3(6)} = 4^{3 \times 6}$$
$$= 4^{18}$$

From Brainstorming 3, it is found that:

$$(3^2)^4 = 3^{2(4)}$$
$$(5^4)^3 = 5^{4(3)}$$
$$(4^3)^6 = 4^{3(6)}$$

In general, $(a^m)^n = a^{mn}$

Brainstorming 4

In pairs

Aim: To determine the value of a number or an algebraic term with a zero index.

Steps:

1. Study and complete the following table.
2. What is your conclusion regarding zero index?

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^3$	$2^{3-3} = 2^0$	$\frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} = 1$	$2^0 = 1$
(d) $m^5 \div m^5$	$m^{5-5} = m^0$	$\frac{\cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m}}{\cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m}} = 1$	$m^0 = 1$
(c) $5^4 \div 5^4$			
(d) $(-7)^2 \div (-7)^2$			
(e) $n^6 \div n^6$			

Discussion:

1. Are your answers similar to the answers of the other groups?
2. What is your conclusion regarding zero index?

From Brainstorming 4, it is found that:

$$2^0 = 1$$
$$m^0 = 1$$

Therefore, a number or an algebraic term with a zero index will give a value of 1.

In general, $a^0 = 1 \; ; a \neq 0$

Brainstorming 5

In groups

Aim: To verify $a^{-n} = \frac{1}{a^n}$.

Steps:

1. Study and complete the following table.

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^5$	$2^{3-5} = 2^{-2}$	$\frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2} = \frac{1}{2^2}$	$2^{-2} = \frac{1}{2^2}$
(b) $m^2 \div m^5$	$m^{2-5} = m^{-3}$	$\frac{m \times m}{m \times m \times m \times m \times m} = \frac{1}{m \times m \times m} = \frac{1}{m^3}$	$m^{-3} = \frac{1}{m^3}$
(c) $3^2 \div 3^6$			
(d) $(-4)^3 \div (-4)^7$			
(e) $p^4 \div p^8$			

Discussion:

1. Are your answers similar to the anwers of the other groups?

2. What is your conclusion?

From Brainstorming 5, it is found that:

$$2^{-2} = \frac{1}{2^2}$$

$$m^{-3} = \frac{1}{m^3}$$

In general, $a^{-n} = \frac{1}{a^n}$; $a \neq 0$