CHAPTER 5

Brainstorming 1

In groups

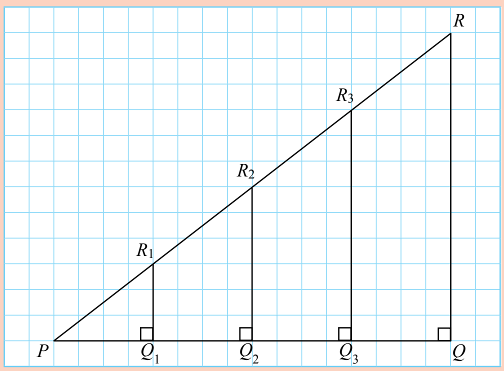
Aim: To identify the relationship between acute angles and the ratios of the sides of right-angled triangles.

Materials: Square grid paper, ruler and pencil.

Steps:

1. Draw a right-angled triangle PQR, where the length PQ is 16 units and the length QR is 12 units.

2. Draw a few straight lines parallel to RQ. Label them as R_1Q_1, R_2Q_2 and R_3Q_3 as shown in the diagram below.



3. Complete the table below with the required measurements.

Acute angle	$\dfrac{\text{Opposite side}}{\text{Hypotenuse}}$	$\dfrac{\text{Adjacent side}}{\text{Hypotenuse}}$	$\dfrac{\text{Opposite side}}{\text{Adjacent side}}$
$\angle QPR$	$\dfrac{R_1Q_1}{PR_1} = \dfrac{3}{5}$	$\dfrac{PQ_1}{PR_1} = \dfrac{4}{5}$	$\dfrac{R_1Q_1}{PQ_1} = \dfrac{3}{4}$
	$\dfrac{R_2Q_2}{PR_2} =$	$\dfrac{PQ_2}{PR_2} =$	$\dfrac{R_2Q_2}{PQ_2} =$
	$\dfrac{R_3Q_3}{PR_3} =$	$\dfrac{PQ_3}{PR_3} =$	$\dfrac{R_3Q_3}{PQ_3} =$
	$\dfrac{RQ}{PR} =$	$\dfrac{PQ}{PR} =$	$\dfrac{RQ}{PQ} =$

Discussion:

1. What is the pattern of your answer to the ratio of the length of the opposite side to the hypotenuse, the ratio of the length of the adjacent side to the hypotenuse and the ratio of the length of the opposite side to the length of the adjacent side?

2. What happens if the size of the angle is changed? Justify your answer.

From Brainstorming 1, it is found that:

Given a fixed acute angle in right-angled triangles of different sizes;

(a) The ratio of the length of the opposite side to the hypotenuse is a constant.

(b) The ratio of the length of the adjacent side to the hypotenuse is a constant.

(c) The ratio of the length of the opposite side to the length of the adjacent side is a constant.

The relationships of the ratios obtained from Brainstorming 1 are trigonometric ratios known as **sine, cosine and tangent**, that is:

$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

Brainstorming 2

In pairs

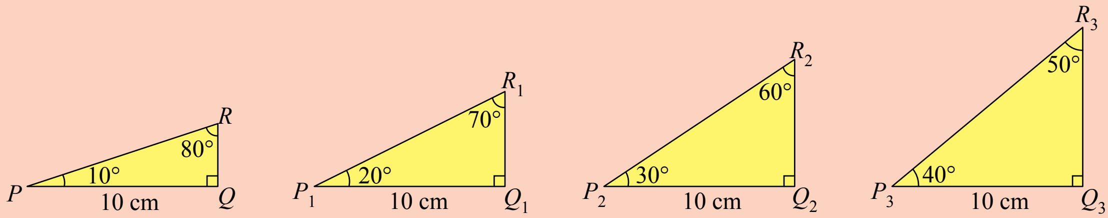
Aim: To identify the impact of changing the size of the angles on the values of sine, cosine and tangent.

Materials: Square grid paper, ruler, protractor and pencil.

Steps:

1. Draw four right-angled triangles as shown below with the base length of 10 cm.

2. Make sure that the angles and lengths of all right-angled triangles are exactly as given.



3. Complete the table below.

sin 10°	sin 20°	sin 30°	sin 40°	sin 50°	sin 60°	sin 70°	sin 80°
$\frac{RQ}{PR}$ $= \frac{1.8}{10.2}$ $= 0.1765$							$\frac{PQ}{PR}$ $= \frac{10}{10.2}$ $= 0.9804$

cos 10°	cos 20°	cos 30°	cos 40°	cos 50°	cos 60°	cos 70°	cos 80°
$\frac{PQ}{PR}$ $= \frac{10}{10.2}$ $= 0.9804$							$\frac{RQ}{PR}$ $= \frac{1.8}{10.2}$ $= 0.1765$

tan 10°	tan 20°	tan 30°	tan 40°	tan 50°	tan 60°	tan 70°	tan 80°
$\frac{RQ}{PQ}$ $= \frac{1.8}{10}$ $= 0.1800$							$\frac{PQ}{RQ}$ $= \frac{10}{1.8}$ $= 5.5556$

Discussion:

1. Based on the values in the table for the trigonometric ratios you have completed, what conclusion can you make?

2. What is your conjecture on

(a) the value of the sine ratio when the angle approaches 0° and 90°?

(b) the value of the cosine ratio when the angle approaches 0° and 90°?

(c) the value of the tangent ratio when the angle approaches 0° and 90°?

From Brainstorming 2, it is found that:

The larger the size of the acute angle

(a) the **larger the value of sine** and its value **approaches 1**.

(b) the **smaller the value of cosine** and its value **approaches zero**.

(c) the **larger the value of tangent**.