CHAPTER 6

Brainstorming 1 In groups

Aim: To verify that angles at the circumference subtended by the same arc are equal.

Materials: A4 paper, compasses, protractor, ruler and pencil.

Steps:

1. Draw a circle of radius 5 cm. Draw a chord PQ (Diagram 1).
2. Draw a chord QR that forms 30° at point Q (Diagram 2). Other groups are encouraged to form acute angles between 20° and 40°.
3. Mark the point S on the circumference and draw chords PS and RS (Diagram 3).
4. Measure $\angle PSR$ and record it in the table below.
5. Repeat step 3 with point T and chords PT and RT (Diagram 4).
6. Measure $\angle PTR$ and record it in the table.

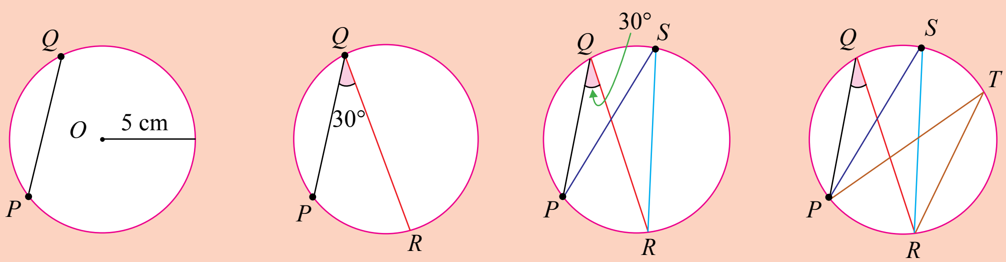


Diagram 1 Diagram 2 Diagram 3 Diagram 4

$\angle PQR$	$\angle PSR$	$\angle PTR$		
30°				

7. You may repeat step 3 with other points on major arc PR. Measure the angle formed and record in the table.
8. Display your group's findings in the Mathematics corner. Give feedback on the findings of other groups.

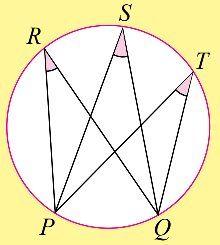
Discussion:

What can you say about the angles at the circumference of the circle subtended by arc PR?

From Brainstorming 1, it is found that:

The angles subtended by arc PR, $\angle PQR$, $\angle PSR$ and $\angle PTR$, are equal.

In general,



Angles at the **circumference subtended** by the **same arc** are **equal**.

$\angle PRQ = \angle PSQ = \angle PTQ$

Brainstorming 2

In pairs

Aim: To verify that angles at the circumference subtended by the same arc are equal.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on the *Compass Tool* to draw a circle (Diagram 1).
2. Click on *Point Tool* and mark three points (Diagram 2).
3. Click on *Text Tool* and label the three points marked in step 2 (Diagram 3).

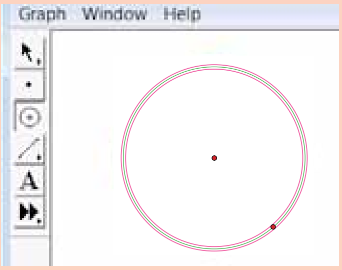


Diagram 1

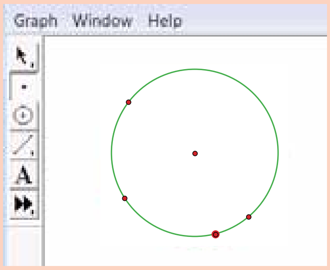


Diagram 2

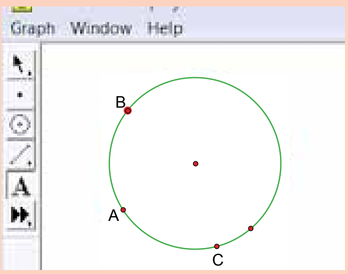


Diagram 3

4. Click on *Straightedge Tool* and draw two straight lines connecting point A and point B as well as point B and point C (Diagram 4).
5. Click on *Selection Arrow Tool* and click on points A, B and C (Diagram 5).
6. Click *Measure* and select *Angle*. The value of $\angle BC$ will be displayed (Diagram 6).

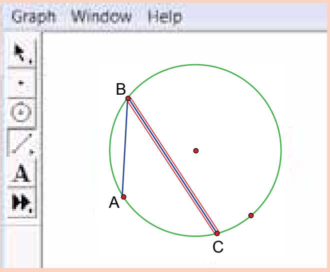


Diagram 4

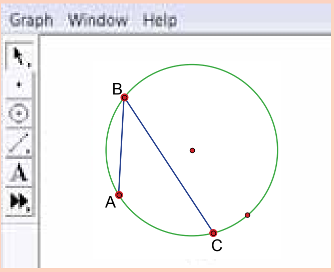


Diagram 5

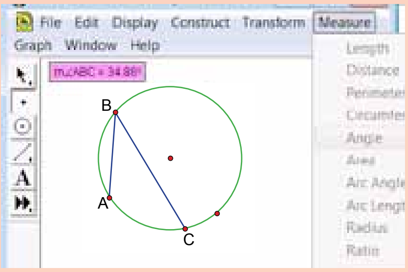


Diagram 6

7. Repeat steps 2 to 4 for point D and step 5 to select points A, D and C (Diagram 7).

8. Repeat step 6. The value of $\angle ADC$ will be displayed (Diagram 8). Notice that the values of $\angle ABC$ and $\angle ADC$ are the same.

9. You can try this with another point on the major arc AC to determine the value of the angle at the circumference.

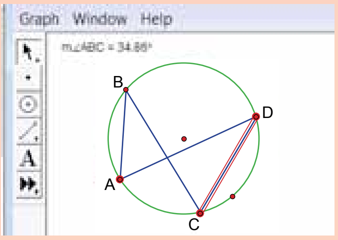


Diagram 7

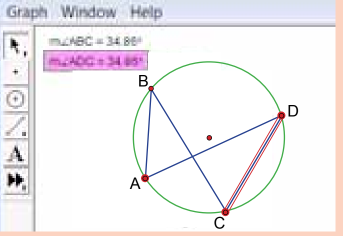


Diagram 8

Discussion:

What can be concluded from your observations in the above activities?

From Brainstorming 2, it is found that:

The angles at the circumference subtended by the same arc are equal.

Brainstorming 3

In groups

Aim:
1. To verify that angles at the circumference subtended by arcs of the same length are equal.
2. To verify that angles at the circumference is proportional to the length of the arc.

Materials: Compasses, protractor, pencil, ruler and A4 paper.

Steps:

1. Draw a circle of radius 5 cm. Without adjusting the gap of the compasses, divide the circumference of circle into six parts (Diagram 1 - Diagram 3).
2. Draw two angles at the circumference that are subtended by two different parts of the same length and label them (Diagram 4).

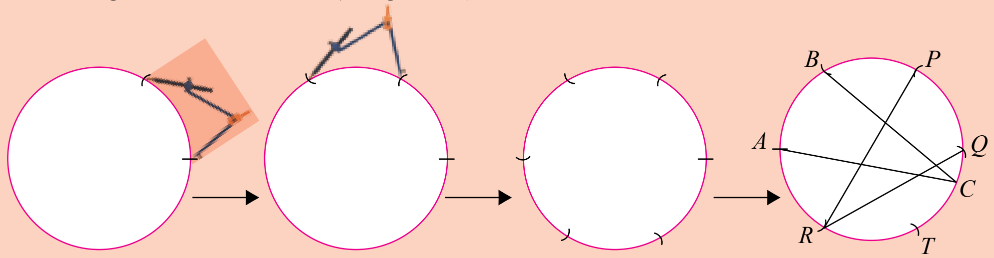


Diagram 1 Diagram 2 Diagram 3 Diagram 4

3. Measure $\angle BCA$ and $\angle PRQ$. Record them in Table 1.
4. Repeat step 1. Draw chords with different arc lengths (Diagram 5). Measure $\angle RPT$ and $\angle BQR$. Record them in Table 2.

Arcs	
BA	PQ
$\angle BCA$	$\angle PRQ$

Table 1

Arcs	
RT	$BR = 2RT$
$\angle RPT$	$\angle BQR$

Table 2

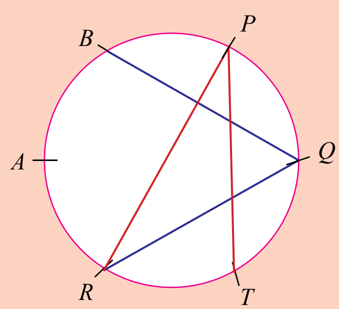


Diagram 5

Discussion:

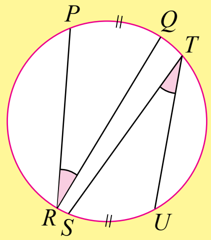
1. What can you conclude about angles at the circumference subtended by arcs of the same length?
2. What is your conclusion on the effects of changing the arc length to the angles subtended at the circumference?

From Brainstorming 3, it is found that:

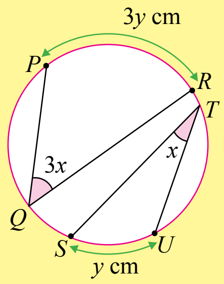
(a) $\angle BCA = \angle PRQ$ [Arc length AB = Arc length PQ].

(b) $\angle BQR = 2 \times \angle RPT$ [Arc length $BR = 2 \times$ Arc length RT].

In general,



Angles at the **circumference** subtended by **arcs of the same length** are equal. If arc length PQ = arc length SU then $\angle PRQ = \angle STU$.



The size of an angle at the **circumference** subtended by an arc is **proportional** to **the arc length**.

Brainstorming 4

In pairs

Aim: To verify the relationship between angles at the centre of a circle and angles at the circumference subtended by the same arc.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on *Compass Tool* to draw a circle.
2. Use *Point Tool* to place three points around its circumference (Diagram 1).
3. Use *Text Tool* to label all points at the circle with A, B, C and centre as D (Diagram 2).
4. Use *Straightedge Tool* to construct lines from one point to another (Diagram 3).

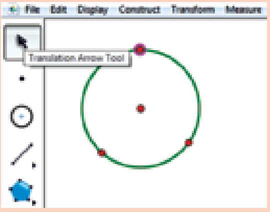


Diagram 1

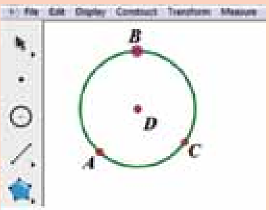


Diagram 2

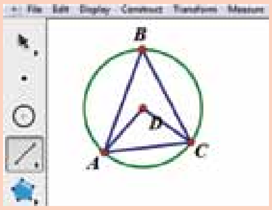


Diagram 3

5. Use *Selection Arrow Tool* to select points A, B and C.
6. Click on the menu *Measure* and select *Angle*. The value of $\angle ABC$ will be displayed.
7. Repeat steps 5 and 6 to get $\angle ADC$. The value of $\angle ADC$ will be displayed (Diagram 4).

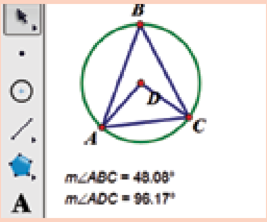


Diagram 4

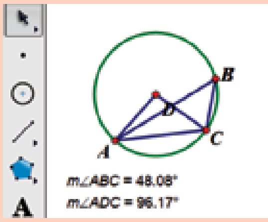


Diagram 5

8. What is the relationship between $\angle ABC$ and $\angle ADC$?
9. Click on point B and move it along the circumference of the circle as shown in Diagram 5. Is the value of $\angle ABC$ still the same as the value obtained in step 6?

Discussion:

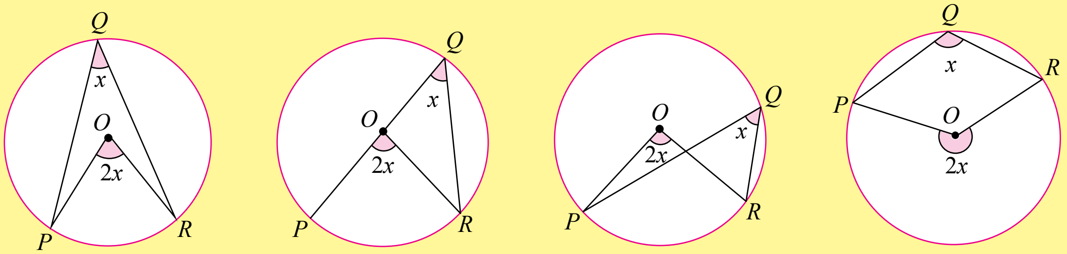
What can you conclude about the relationship between angles at the centre of a circle and angles at the circumference of a circle subtended by the same arc?

From Brainstorming 4, it is found that:

(a) $\angle ADC = 2 \times \angle ABC$

(b) The value of $\angle ABC$ is constant even though point B is moved along the circumference of the circle.

In general,



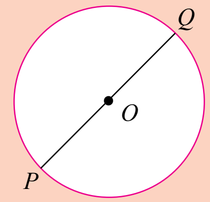
The size of the **angle** at the **centre of a circle** (central angle) subtended by the same arc is **twice** the size of angle at the **circumference**.

Brainstorming 5

In pairs

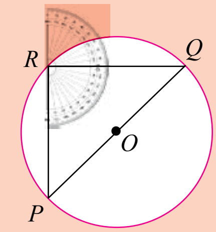
Aim: To determine the angles subtended by the diameter.

Materials: Compasses, protractor, pencil, ruler and drawing paper.



Steps:

1. Draw a circle with centre O and diameter PQ as in the diagram.
2. Draw two chords, PR and QR as in the diagram. Measure the value of $\angle PRQ$.
3. Change the position of point R at the circumference of the circle. Measure the new value of $\angle PRQ$.



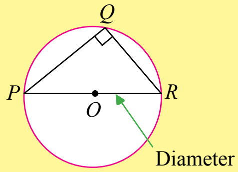
Discussion:

1. What can you conclude about the value of $\angle PRQ$ when the position of point R is changed at the circumference?
2. What is the value of the angle at the circumference of a circle subtended by the diameter?

From Brainstorming 5, it is found that:

For all positions of point R at the circumference of the circle subtended by diameter PQ, the value of $\angle PRQ$ is 90°.

In general,



The angle at the circumference of circle subtended by the diameter is 90°.
If PQR is a semicircle, then $\angle PQR = 90°$.

Brainstorming 6

In pairs

Aim: To determine the relationship between opposite interior angles of a cyclic quadrilateral.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on *Compass Tool* to draw a circle.
2. Click on *Straightedge Tool* to construct four lines from one point to another point on its circumference (Diagram 1).
3. Use *Text Tool* to label all points connecting the line with A, B, C and D.
4. Use *Selection Arrow Tool* to select D, A, and B.
5. Click on the menu *Measure* and select *Angle*. The value of $\angle DAB$ will be displayed.
6. Repeat steps 4 and 5 to get $\angle ABC$, $\angle BCD$ and $\angle CDA$ (Diagram 2).

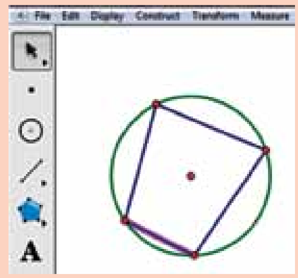


Diagram 1

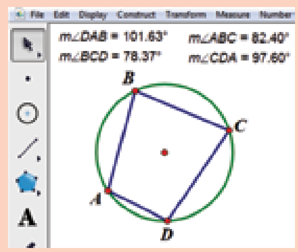


Diagram 2

Discussion:

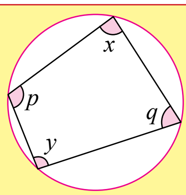
1. What are the relationships between $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle ADC$?
2. What can you conclude about the relationships between the angles of a cyclic quadrilateral?

From Brainstorming 6, it is found that:

(a) $\angle DAB + \angle BCD = 180°$ and $\angle ABC + \angle ADC = 180°$

(b) **The sum of the opposite** interior angles in a cyclic quadrilateral is 180°.

In general,



The sum of opposite interior angles in a cyclic quadrilateral is 180°.
$\angle x + \angle y = 180°$ and $\angle p + \angle q = 180°$

Brainstorming 7

In pairs

Aim: To measure the angle between tangent and radius of a circle at the point of tangency.

Materials: Dynamic software

Steps:

1. Start with *New Sketch* and click on the *Compass Tool* to draw a circle (Diagram 1).
2. Click on *Straightedge Tool* to draw a straight line from the centre of the circle to a point on the circumference (Diagram 2).
3. Click on *Arrow Tool* to select point on the circumference and straight line.
4. Click *Construct* and select *Perpendicular Line* (Diagram 3).
5. Use *Point Tool* to mark the points and label them with the *Text tool* as *A*, *B* and *C* (Diagram 4).
6. Use *Selection Arrow Tool* to select *A*, *B* and *C*.

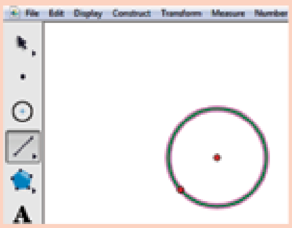


Diagram 1

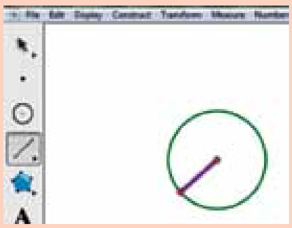


Diagram 2

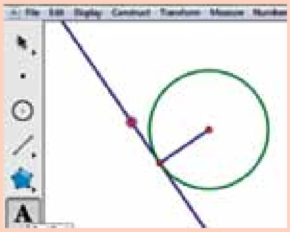


Diagram 3

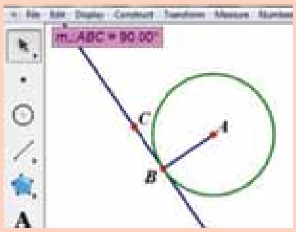


Diagram 4

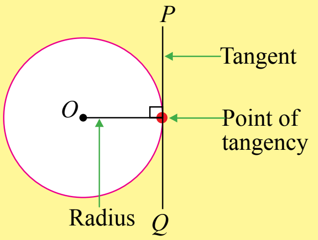
7. Click on the menu *Measure* and select *Angle*. The value of *ABC* will be displayed.
8. Repeat step 2 to step 7 to draw tangent lines on the other side of the circle and determine the angle between tangent and radius at the point of tangency.

Discussion:
What conclusions can you draw about the value of the angle between tangent and radius at the point of tangency?

From Brainstorming 7, it is found that:

When tangent and radius intersect at the point of tangency, a right angle is formed. Thus $\angle ABC = 90°$.

In general,



The radius of a circle that intersects with tangent to the circle at the point of tangency will form a 90° angle with the tangent.

Brainstorming 8

In pairs

Aim: To determine the properties related to two tangents to a circle.

Materials: Drawing paper, compasses, protractor, ruler and pencil.

Steps:

1. Draw a circle of radius 3 cm with centre O. Draw a straight line 8 cm from the centre O and label as OA (Diagram 1).
2. Draw another circle of radius 7 cm with point A as centre of the circle. Mark the intersection points of both circles as B and C (Diagram 2).
3. Draw straight lines OB, OC, AB and AC (Diagram 3).

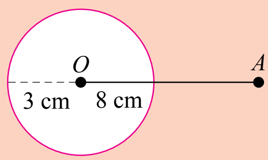


Diagram 1

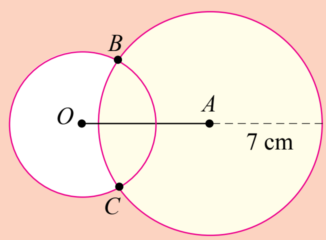


Diagram 2

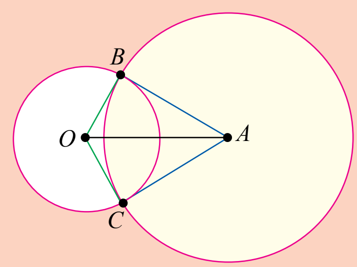


Diagram 3

4. Measure the following and complete the table below.

$\angle AOB$	$\angle AOC$	$\angle OBA$	$\angle OCA$	$\angle OAB$	$\angle OAC$	Length			
						OB	OC	AB	AC

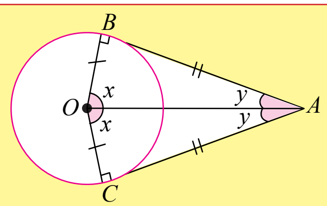
5. Display your group's findings in the Mathematics corner. Compare your group's answers with other groups.

Discussion:
What are your conclusions regarding the pairs of $\angle AOB$ and $\angle AOC$, $\angle OBA$ and $\angle OCA$, $\angle OAB$ and $\angle OAC$ and also the length of lines OB, OC, AB and AC?

From Brainstorming 8, it is found that:

(a) $\angle AOB = \angle AOC$, $\angle OBA = \angle OCA$ and $\angle OAB = \angle OAC$
(b) Length of OB = length of OC and length of AB = length of AC

In general,



If two tangents to a circle with centre O and points of tangency B and C meet at point A, then,

- $BA = CA$
- $\angle BOA = \angle COA$
- $\angle OAB = \angle OAC$