KURIKULUM STANDARD SEKOLAH MENENGAH

MATHEMATICS
FORM 3

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Contents

Introduction

This Form 3 Mathematics Textbook is prepared based on *Kurikulum Standard Sekolah Menengah (KSSM)*. This book contains 9 chapters arranged systematically based on Form 3 Mathematics *Dokumen Standard Kurikulum dan Pentaksiran (DSKP)*.

At the beginning of each chapter, students are introduced to stimulating materials related to daily life to stimulate their thinking about the topic. In addition, Learning Standard and word list also give a visual summary about the chapter's content.

This book contains the following special features:

	Description
What will you learn?	Contains learning standard that students will learn in each chapter.
Why do you learn this chapter?	Applications of knowledge in this chapter in related career fields.
Exploring Era	History of ancient academy or original exploration of the chapter in Mathematics.
WORD BANK	Word list contained in each chapter.
Brainstorming Individual In pairs In groups	Helps students to understand the basic mathematical concept via individual, pair or group activities.
BULLETIN	Gives additional information about the chapter learned.
QUIZ	Questions that test students' capability to understand certain technique in each chapter.
REMINDER	Grabs students' attention to additional facts that need to be reminded of, mistakes that students commonly make, and carelessness to be avoided.
TIPS	Exposes students to additional knowledge that they need to know.
SMART MIND	Presents mind-stimulating questions for enhancement of students' critical and creative thinking.

	Description
SMART TECHNOLOGY	Exposes students to the use of technological tools in the learning of mathematics.
DISCUSSION CORNER	Develops communication skills mathematically.
FLASHBACK	Helps students to remember what they have learnt.
SMART FINGER	Shows the use of scientific calculators in calculations.
PROJECT	Enables students to carry out assignments and then present their completed work in class.
MIND TEST	Test students' understanding on the concepts they have learnt.
	Indicates HOTS questions to help in developing students' higher order thinking skills.
Dynamic Challenge	Prepares more diversified exercises which incorporate the elements of LOTS, HOTS, TIMSS and PISA assessment.
	Enables students to scan QR Code using mobile device.
EXPLORING MATHEMATICS	Covers applicable concepts of digital tool calculators, hands on activities and games that aim to provides additional activities to effectively enhance students' understanding.
CONCEPT MAP	Overall chapter summary that students learnt.
SELF-REFLECT	Looks back whether students have achieved the learning standard.
Checking Answers	Checks answers with alternative methods.
STEM	Activities with elements of Science, Technology, Engineering and Mathematics.

Symbols and Formulae

SYMBOLS

$\sqrt{\ }$	root
π	pi
$a : b$	ratio of a to b
$A \times 10^n$	standard form where $1 \leqslant A < 10$ and n is an integer
$=$	is equal to
$\approx$	is approximately equal to
$\neq$	is not equal to
$>$	is more than
$\geqslant$	is more than or equal to
$<$	is less than
$\leqslant$	is less than or equal to
Δ	triangle
$\angle$	angle
$^\circ$	degree
$'$	minute
$''$	second

FORMULAE

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$I = Prt$

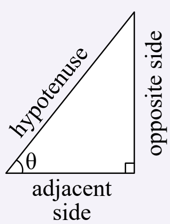
$MV = P(1 + \frac{r}{n})^{nt}$

$A = P + Prt$

$\sin \theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

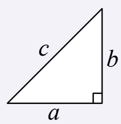
$\cos \theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan \theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$



$\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

Pythagorean theorem:



$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$a^2 = c^2 - b^2$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Gradient, $m = \dfrac{\text{vertical distance}}{\text{horizontal distance}}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = -\dfrac{y\text{-intercept}}{x\text{-intercept}}$

http://bukuteksksssm.my/Mathematics/F3/Index.html

Download the free *QR Code* scanner to your mobile devices. Scan *QR Code* or visit the website http://bukuteksksssm.my/Mathematics/F3/Index.html to download files for brainstorming. Then, save the downloaded file for offline use.

Note: Students can download free *GeoGebra and Geometer's Sketchpad* (*GSP*) software to open related files.

CHAPTER 1

Indices

What will you learn?

- 1.1 Index Notation
- 1.2 Law of Indices

Why do you learn this chapter?

- Writing a number in index notation enables the number stated in a simple and easily understood form. Various operations of mathematics that involve numbers in index notation can be performed by using laws of indices.
- Concept of index is used in the fields of science, engineering, accounting, finance, astronomy, computer and so on.

Kenyir Lake, located in the district of Hulu Terengganu, in Terengganu, is the biggest man-made lake in Southeast Asia. Kenyir Lake is a world famous tourist destination known for its unique natural beauty. Kenyir Lake is an important water catchment area. Kenyir Lake, which was built in the year 1985, supplies water to Sultan Mahmud Power Station. The estimated catchment area at the main dam is 2 600 km^2 with a reservoir volume of 13 600 million cubic metre. During rainy season, the volume of water in the catchment area will increase sharply. What action should be taken to address this situation?

Exploring Era

Index notation is an important element in the development of mathematics and computer programming. The use of positive indices was introduced by Rene Descartes (1637), a well-known French mathematician. Sir Isaac Newton, another well-known British mathematician, developed the field of index notation and introduced negative indices and fractional indices.

http://bukuteksksm.my/Mathematics/F3/ExploringEraChapter1.pdf

WORD BANK

• base	• *asas*
• factor	• *faktor*
• index	• *indeks*
• fractional index	• *indeks pecahan*
• power	• *kuasa*
• root	• *punca kuasa*
• index notation	• *tatatanda indeks*

1.1 Index Notation

What is repeated multiplication in index form?

The development of technology not only makes most of our daily tasks easier, it also saves cost of expenses in various fields. For instance, the use of memory cards in digital camera enables users to store photographs in a large number and to delete or edit unsuitable photographs before printing.

LEARNING STANDARD

Represent repeated multiplication in index form and describe its meaning.

DISCUSSION CORNER

Discuss the value of the capacity of a pen drive.

BULLETIN

The nuclear fission of uranium U-320 follows the pattern 30, 31, 32, …

In the early stage, memory cards were made with a capacity of 4MB. The capacity was increased with time and the needs of users. Did you know that the value of capacity of memory cards is calculated using a special form that is 2^n?

In Form 1, you have learnt that $4^3 = 4 \times 4 \times 4$. The number 4^3 is written in index notation, 4 is the **base** and 3 is the **index** or **exponent**. The number is read as '4 to the power of 3'.

Hence, a number in index notation or in index form can be written as;

a^n ← Index
a ← Base

You have also learnt that $4^2 = 4 \times 4$ and $4^3 = 4 \times 4 \times 4$. For example;

$\underbrace{4 \times 4}_{\text{Repeated two times}} = 4^{2}$

- 4^2 → The value of index is 2
- Repeated two times → The value of index is the same as the number of times 4 is multiplied repeatedly.

$\underbrace{4 \times 4 \times 4}_{\text{Repeated three times}} = 4^{3}$

- 4^3 → The value of index is 3
- Repeated three times → The value of index is the same as the number of times 4 is multiplied repeatedly.

Example 1

Write the following repeated multiplications in index form a^n.

(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

(b) $0.3 \times 0.3 \times 0.3 \times 0.3$

(c) $(-2) \times (-2) \times (-2)$

(d) $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

(e) $m \times m \times m \times m \times m \times m \times m$

(f) $n \times n \times n \times n \times n \times n \times n \times n$

REMINDER

$2^5 \neq 2 \times 5$ $4^3 \neq 4 \times 3$

$a^n \neq a \times n$

Solution:

(a) $\underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{repeated six times}} = 5^6$

(b) $\underbrace{0.3 \times 0.3 \times 0.3 \times 0.3}_{\text{repeated four times}} = (0.3)^4$

(c) $\underbrace{(-2) \times (-2) \times (-2)}_{\text{repeated three times}} = (-2)^3$

(d) $\underbrace{\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}}_{\text{repeated five times}} = \left(\frac{1}{4}\right)^5$

(e) $\underbrace{m \times m \times m \times m \times m \times m \times m}_{\text{repeated seven times}} = m^7$

(f) $\underbrace{n \times n \times n \times n \times n \times n \times n \times n}_{\text{repeated eight times}} = n^8$

From the solution in Example 1, it is found that the value of index in an index form is the same as the number of times the base is multiplied repeatedly. In general,

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}} \; ; \; a \neq 0$$

MIND TEST 1.1a

1. Complete the following table with base or index for the given numbers or algebraic terms.

5^3 $\quad (-4)^7$

$\left(\frac{1}{2}\right)^{10}$ $\quad m^6$ $\quad \left(-\frac{3}{7}\right)^4$

n^0 $\quad (0.2)^9$

x^{20} $\quad \left(2\frac{1}{3}\right)^2$

8

Base	Index
5	
	7
$\frac{1}{2}$	
	6
n	
	9
	4
x	
	2
8	

2. State the following repeated multiplications in index form a^n.

(a) $6 \times 6 \times 6 \times 6 \times 6 \times 6$

(b) $0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$

(c) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

(d) $(-m) \times (-m) \times (-m) \times (-m) \times (-m)$

(e) $1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3}$

(f) $\left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right) \times \left(-\frac{1}{n}\right)$

3. Convert the numbers or algebraic terms in index form into repeated multiplications.

(a) $(-3)^3$

(b) $(2.5)^4$

(c) $\left(\frac{2}{3}\right)^5$

(d) $\left(-2\frac{1}{4}\right)^3$

(e) k^6

(f) $(-p)^7$

(g) $\left(\frac{1}{m}\right)^8$

(h) $(3n)^5$

How do you convert a number into a number in index form?

LEARNING STANDARD

Rewrite a number in index form and vice versa.

A number can be written in index form if a suitable base is selected. You can use repeated division method or repeated multiplication method to convert a number into a number in index form.

FLASHBACK

$4 \times 4 \times 4 = 4^3$

Example 2

Write 64 in index form using base of 2, base of 4 and base of 8.

Solution:

Repeated Division Method

(a) Base of 2

- 64 is divided repeatedly by 2.

$$n = 6 \left\{ \begin{array}{r|r} 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \right.$$

Hence, $64 = 2^6$

(b) Base of 4

- 64 is divided repeatedly by 4.

$$n = 3 \left\{ \begin{array}{r|r} 4 & 64 \\ 4 & 16 \\ 4 & 4 \\ & 1 \end{array} \right.$$

Hence, $64 = 4^3$

(c) Base of 8

- 64 is divided repeatedly by 8.

$$n = 2 \left\{ \begin{array}{r|r} 8 & 64 \\ 8 & 8 \\ & 1 \end{array} \right.$$

Hence, $64 = 8^2$

The division is continued until 1 is obtained.

Repeated Multiplication Method

(a) Base of 2

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

4

8

16

32

64

Hence, $64 = 2^6$

(b) Base of 4

$4 \times 4 \times 4$

16

64

Hence, $64 = 4^3$

(c) Base of 8

$8 \times 8 = 64$

Hence, $64 = 8^2$

DISCUSSION CORNER

Which of the repeated division method and the repeated multiplication method is easier to convert a number into a number in index form? Discuss.

Example 3

Write $\dfrac{32}{3\,125}$ in index form using base of $\dfrac{2}{5}$.

Solution:

Repeated Division Method

$$n = 5 \left\{ \begin{array}{r|r} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ 2 & 2 \\ & 1 \end{array} \right. \qquad n = 5 \left\{ \begin{array}{r|r} 5 & 3\,125 \\ 5 & 625 \\ 5 & 125 \\ 5 & 25 \\ 5 & 5 \\ & 1 \end{array} \right.$$

Hence, $\dfrac{32}{3\,125} = \left(\dfrac{2}{5}\right)^5$

Repeated Multiplication Method

$$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$$

$$\frac{4}{25}$$

$$\frac{8}{125}$$

$$\frac{16}{625}$$

$$\frac{32}{3\,125}$$

Hence, $\dfrac{32}{3\,125} = \left(\dfrac{2}{5}\right)^5$

MIND TEST 1.1b

1. Write each of the following numbers in index form using the stated base in brackets.

(a) 81 [base of 3] (b) 15 625 [base of 5] (c) $\dfrac{64}{125}$ $\left[\text{base of } \dfrac{4}{5}\right]$

(d) 0.00032 [base of 0.2] (e) – 16 384 [base of (– 4)] (f) $\dfrac{1}{16}$ $\left[\text{base of } \left(-\dfrac{1}{4}\right)\right]$

How do you determine the value of the number in index form , a^n?

The value of a^n can be determined by repeated multiplication method or using a scientific calculator.

Example 4

Calculate the values of the given numbers in index form.

(a) 2^5

$$2 \times 2 \times 2 \times 2 \times 2$$
$$4 \times 2$$
$$8 \times 2$$
$$16 \times 2$$
$$32$$

Hence, $2^5 = 32$

(b) $(0.6)^3$

$$0.6 \times 0.6 \times 0.6$$
$$0.36 \times 0.6$$
$$0.216$$

$0.6^3 = 0.216$

Hence, $0.6^3 = 0.216$

QUIZ

$(m)^4 = 16$
What are the possible values of m?

Example 5

(a) $5^4 = 625$ → 5 ^ 4 =

(b) $(-7)^3 = -343$ → ((–) 7) ^ 3 =

(c) $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$ → (2 ab/c 3) ^ 4 =

(d) $\left(1\frac{3}{5}\right)^2 = \frac{64}{25}$ → (1 ab/c 3 ab/c 5) ^ 2 =

(e) $(-0.5)^6 = 0.015625$ → ((–) 0 . 5) ^ 6 =

SMART FINGER

REMINDER

Negative or fractional base must be placed within brackets when using a calculator to calculate values of given numbers.

DISCUSSION CORNER

Calculate questions (c), (d) and (e) in Example 5 without using brackets. Are the answers the same? Discuss.

MIND TEST 1.1c

1. Calculate the value of each of the following numbers in index form.

(a) 9^4 (b) $(-4)^5$ (c) $(2.5)^3$ (d) $(-3.2)^3$

(e) $\left(\frac{3}{8}\right)^5$ (f) $\left(-\frac{1}{6}\right)^4$ (g) $\left(1\frac{2}{3}\right)^2$ (h) $\left(-2\frac{1}{3}\right)^3$

1.2 Law of Indices

What is the relationship between multiplication of numbers in index form with the same base and repeated multiplication?

LEARNING STANDARD

Relate the multiplication of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.

Brainstorming 1 In pairs

Aim: To identify the relationship between multiplication of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Exhibit three examples in the mathematics corner for other groups to give feedback.

Multiplication of numbers in index form	Repeated multiplication
(a) $2^3 \times 2^4$	$\underbrace{(2 \times 2 \times 2)}_{\text{3 factors}} \times \underbrace{(2 \times 2 \times 2 \times 2)}_{\text{4 factors}} = \underbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}_{\text{7 factors (overall)}} = 2^7$ $2^3 \times 2^4 = 2^{\boxed{7}}$ $2^3 \times 2^4 = 2^{\boxed{3+4}}$ $\boxed{7 = 3 + 4}$
(b) $3^2 \times 3^3$	$\underbrace{(3 \times 3)}_{\text{2 factors}} \times \underbrace{(3 \times 3 \times 3)}_{\text{3 factors}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3}_{\text{5 factors (overall)}} = 3^5$ $3^2 \times 3^3 = 3^{\boxed{}}$ $3^2 \times 3^3 = 3^{\boxed{}}$

Multiplication of numbers in index form	Repeated multiplication
(c) $5^4 \times 5^2$	$\underbrace{(5 \times 5 \times 5 \times 5)}_{\text{4 factors}} \times \underbrace{(5 \times 5)}_{\text{2 factors}} = \underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{6 factors (overall)}} = 5^6$ $5^4 \times 5^2 = 5^{\square}$ $5^4 \times 5^2 = 5^{\square}$

Discussion:

What is your conclusion regarding the relationship between multiplication of numbers in index form and repeated multiplication?

From Brainstorming 1, it is found that;

$$2^3 \times 2^4 = 2^{3+4}$$
$$3^2 \times 3^3 = 3^{2+3}$$
$$5^4 \times 5^2 = 5^{4+2}$$

In general, $\boxed{a^m \times a^n = a^{m+n}}$

DISCUSSION CORNER

Given,
$a^m \times a^n = b^m \times b^n$.
Is $a = b$? Discuss.

Example 6

Simplify each of the following.

(a) $7^2 \times 7^3$ (b) $(0.2)^2 \times (0.2)^4 \times (0.2)^5$ (c) $2k^2 \times 4k^3$ (d) $3m^4 \times \frac{1}{6}m^5 \times 12m$

Solution:

(a) $7^2 \times 7^3$
$= 7^{2+3}$
$= 7^5$

(b) $(0.2)^2 \times (0.2)^4 \times (0.2)^5$
$= (0.2)^{2+4+5}$
$= (0.2)^{11}$

REMINDER

$a = a^1$

(c) $2k^2 \times 4k^3$
$= \underbrace{(2 \times 4)}_{\text{Operation of the coefficients}}(k^2 \times k^3)$
$= 8k^{2+3}$
$= 8k^5$

(d) $3m^4 \times \frac{1}{6}m^5 \times 12m$
$= (3 \times \frac{1}{6} \times 12)\,(m^4 \times m^5 \times m^1)$
$= 6m^{4+5+1}$
$= 6m^{10}$

SMART MIND

If $m^a \times m^b = m^8$, such that $a > 0$ and $b > 0$, what are the possible values of a and b?

MIND TEST 1.2a

1. Simplify each of the following.

(a) $3^2 \times 3 \times 3^4$

(b) $(-0.4)^4 \times (-0.4)^3 \times (-0.4)$

(c) $\left(\frac{4}{7}\right) \times \left(\frac{4}{7}\right)^3 \times \left(\frac{4}{7}\right)^5$

(d) $\left(-1\frac{2}{5}\right)^2 \times \left(-1\frac{2}{5}\right)^3 \times \left(-1\frac{2}{5}\right)^5$

(e) $4m^2 \times \frac{1}{2}m^3 \times (-3)m^4$

(f) $n^6 \times \frac{4}{25}n^2 \times \frac{5}{4}n^3 \times n$

(g) $-x^4 \times \frac{25}{4}x \times \frac{12}{5}x^2$

(h) $-\frac{1}{2}y^5 \times (-6)y^3 \times \frac{1}{3}y^4$

CHAPTER 1

How do you simplify a number or an algebraic term in index form with different bases?

TIPS

Group the numbers or algebraic terms with the same base first. Then add the indices for the terms with the same base.

Example 7

Simplify each of the following.

(a) $m^3 \times n^2 \times m^4 \times n^5$

(b) $(0.3)^2 \times (0.2)^2 \times 0.3 \times (0.2)^5 \times (0.3)^3$

(c) $p^2 \times m^3 \times p^4 \times n^3 \times m^4 \times n^2$

(d) $-m^4 \times 2n^5 \times 3m \times \frac{1}{4}n^2$

Solution:

(a) $m^3 \times n^2 \times m^4 \times n^5$

$= m^3 \times m^4 \times n^2 \times n^5$ ← Group the terms with the same base.

$= m^{3+4} \times n^{2+5}$ ← Add the indices for terms with the same base.

$= m^7 \times n^7$

$= m^7n^7$

(b) $(0.3)^2 \times (0.2)^2 \times 0.3 \times (0.2)^5 \times (0.3)^3$

$= (0.3)^2 \times (0.3)^1 \times (0.3)^3 \times (0.2)^2 \times (0.2)^5$

$= (0.3)^{(2+1+3)} \times (0.2)^{(2+5)}$

$= (0.3)^6 \times (0.2)^7$

(c) $p^2 \times m^3 \times p^4 \times n^3 \times m^4 \times n^2$

$= m^3 \times m^4 \times n^3 \times n^2 \times p^2 \times p^4$

$= m^{3+4} \times n^{3+2} \times p^{2+4}$

$= m^7n^5p^6$

(d) $-m^4 \times 2n^5 \times 3m \times \frac{1}{4}n^2$

$= (-1 \times 2 \times 3 \times \frac{1}{4})\, m^4 \times m^1 \times n^5 \times n^2$

$= -\frac{3}{2}m^{4+1}n^{5+2}$

$= -\frac{3}{2}m^5n^7$

REMINDER

$-a^n \neq (-a)^n$

Example:
$-3^2 \neq (-3)^2$
$-9 \neq 9$

MIND TEST 1.2b

1. State in simplest index form.

(a) $5^4 \times 9^3 \times 5 \times 9^2$

(b) $(0.4)^2 \times (1.2)^3 \times (0.4) \times (1.2)^5 \times (1.2)$

(c) $12x^5 \times y^3 \times \frac{1}{2}x \times \frac{2}{3}y^4$

(d) $-2k^5 \times p^6 \times \frac{1}{4}p^5 \times 3k$

What is the relationship between division of numbers in index form with the same base and repeated multiplication?

LEARNING STANDARD

Relate the division of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.

Brainstorming 2

In pairs

Aim: To identify the relationship between division of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your findings.

Division of numbers in index form	Repeated multiplication
(a) $4^5 \div 4^2$	$\frac{4^5}{4^2} = \frac{\overbrace{\cancel{4}\times\cancel{4}\times 4\times 4\times 4}^{5\text{ factors}}}{\underbrace{\cancel{4}\times\cancel{4}}_{2\text{ factors}}} = \underbrace{4\times 4\times 4}_{3\text{ factors (Remainder)}} = 4^3$ $4^5 \div 4^2 = 4^{\boxed{3}}$ $4^5 \div 4^2 = 4^{\boxed{5-2}}$ $\boxed{3 = 5-2}$
(b) $2^6 \div 2^2$	$\frac{2^6}{2^2} = \frac{\overbrace{\cancel{2}\times\cancel{2}\times 2\times 2\times 2\times 2}^{6\text{ factors}}}{\underbrace{\cancel{2}\times\cancel{2}}_{2\text{ factors}}} = \underbrace{2\times 2\times 2\times 2}_{4\text{ factors (Remainder)}} = 2^4$ $2^6 \div 2^2 = 2^{\boxed{}}$ $2^6 \div 2^2 = 2^{\boxed{}}$
(c) $(-3)^5 \div (-3)^3$	$\frac{(-3)^5}{(-3)^3} = \frac{\overbrace{(\cancel{-3})\times(\cancel{-3})\times(\cancel{-3})\times(-3)\times(-3)}^{5\text{ factors}}}{\underbrace{(\cancel{-3})\times(\cancel{-3})\times(\cancel{-3})}_{3\text{ factors}}} = \underbrace{(-3)\times(-3)}_{2\text{ factors (Remainder)}} = (-3)^2$ $(-3)^5 \div (-3)^3 = (-3)^{\boxed{}}$ $(-3)^5 \div (-3)^3 = (-3)^{\boxed{}}$

Discussion:

What is the relationship between division of numbers in index form and repeated multiplication?

From Brainstorming 2, it is found that;

$4^5 \div 4^2 = 4^{5-2}$
$2^6 \div 2^2 = 2^{6-2}$
$(-3)^5 \div (-3)^3 = (-3)^{5-3}$

In general, $\boxed{a^m \div a^n = a^{m-n}}$

SMART MIND

Given $m^{a-b} = m^7$ and $0 \leqslant a \leqslant 10$. If $a > b$, state the possible values of a and b.

Example 8

Simplify each of the following.

(a) $5^4 \div 5^2$
(b) $(-3)^4 \div (-3)^2 \div (-3)$
(c) $m^4n^3 \div m^2n$
(d) $25x^2y^3 \div 5xy$
(e) $12m^{10} \div 4m^5 \div m^2$
(f) $-16p^8 \div 2p^5 \div 4p^2$

Solution:

(a) $5^4 \div 5^2$
$= 5^{4-2}$
$= 5^2$

(b) $(-3)^4 \div (-3)^2 \div (-3)$
$= (-3)^4 \div (-3)^2 \div (-3)^1$
$= (-3)^{4-2-1}$
$= (-3)^1$
$= -3$

(c) $m^4n^3 \div m^2n$
$= m^4n^3 \div m^2n^1$
$= m^{4-2}\,n^{3-1}$
$= m^2\,n^2$

(d) $25x^2y^3 \div 5xy$

$= 25x^2y^3 \div 5x^1y^1$

$= \frac{25}{5} x^{2-1} y^{3-1}$ Operation of the coefficients

$= 5x^1y^2$

$= 5xy^2$

(e) $12m^{10} \div 4m^5 \div m^2$

$= \frac{12}{4}(m^{10} \div m^5 \div m^2)$

$= 3(m^{10-5}) \div m^2$

$= 3m^{5-2}$

$= 3m^3$

(f) $-16p^8 \div 2p^5 \div 4p^2$

$= \frac{-16}{2}(p^8 \div p^5) \div 4p^2$

$= -8p^{8-5} \div 4p^2$

$= -8p^3 \div 4p^2$

$= -\frac{8}{4}(p^3 \div p^2)$

$= -2p^{3-2}$

$= -2p^1$

$= -2p$

MIND TEST 1.2c

1. Simplify each of the following.

(a) $4^5 \div 4^4$ (b) $7^{10} \div 7^6 \div 7^2$ (c) $\dfrac{m^8n^6}{m^4n}$

(d) $\dfrac{27x^4y^5}{9x^3y^2}$ (e) $m^7 \div m^2 \div m^4$ (f) $-25h^4 \div 5h^2 \div h$

2. Copy and complete each of the following equations.

(a) $8^{\square} \div 8^4 \div 8^3 = 8$ (b) $m^4n^{\square} \div m^{\square}\, n^5 = m^2n$

(c) $\dfrac{m^{10}\, n^4 \times m^{\square}\, n^2}{m^7n} = m^5n^{\square}$ (d) $\dfrac{27x^3y^6 \times xy^{\square}}{\square x^2y^3} = 3x^{\square}\, y^5$

3. If $\dfrac{2^x \times 3^y}{2^4 \times 3^2} = 6$, determine the value of $x + y$.

What is the relationship between a number in index form raised to a power and repeated multiplication?

LEARNING STANDARD

Relate the numbers in index form raised to a power, to repeated multiplication, and hence make generalisation.

Brainstorming 3

In pairs

Aim: To identify the relationship between a number in index form raised to a power and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).

2. Discuss with your friend and state three other examples.

3. Present your finding.

Index form raised to a power	Repeated multiplication in index form	Conclusion
(a) $(3^2)^4$	$\overbrace{3^2 \times 3^2 \times 3^2 \times 3^2}^{4\text{ factors}}$ $= 3^{\underbrace{2+2+2+2}_{4\text{ times}}}$ 2 is added 4 times $= 3^{2(4)}$	$(3^2)^4 = 3^{2(4)}$ $= 3^8$

Index form raised to a power	Repeated multiplication in index form	Conclusion
(b) $(5^4)^3$	$\overbrace{5^4 \times 5^4 \times 5^4}^{\text{3 factors}}$ $= 5^{\underbrace{4+4+4}_{\text{3 times}}}$ $= 5^{4(3)}$ 4 is added 3 times	$(5^4)^3 = 5^{\boxed{}}$ $= 5^{\boxed{}}$
(c) $(4^3)^6$	$\overbrace{4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3}^{\text{6 factors}}$ $= 4^{\underbrace{3+3+3+3+3+3}_{\text{6 times}}}$ $= 4^{3(6)}$ 3 is added 6 times	$(4^3)^6 = 4^{\boxed{}}$ $= 4^{\boxed{}}$

Discussion:

What is your conclusion regarding the index form raised to a power and repeated multiplication in index form?

The conclusion in Brainstorming 3 can be checked using the following method.

Example (a)

$$(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2} = 3^8$$
$$3^{2(4)} = 3^{2 \times 4} = 3^8$$

Example (b)

$$(5^4)^3 = 5^4 \times 5^4 \times 5^4 = 5^{4+4+4} = 5^{12}$$
$$5^{4(3)} = 5^{4 \times 3} = 5^{12}$$

Example (c)

$$(4^3)^6 = 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 = 4^{3+3+3+3+3+3} = 4^{18}$$
$$4^{3(6)} = 4^{3 \times 6} = 4^{18}$$

From Brainstorming 3, it can be found that;

$$(3^2)^4 = 3^{2(4)}$$
$$(5^4)^3 = 5^{4(3)}$$
$$(4^3)^6 = 4^{3(6)}$$

In general, $(a^m)^n = a^{mn}$

SMART MIND

Given,

$m^{rt} = 3^{12}$

What are the possible values of m, r and t if $r > t$?

Example 9

1. Simplify each of the following.

(a) $(3^4)^2$ (b) $(h^3)^{10}$ (c) $((-y)^6)^3$

2. Determine whether the following equations are **true** or **false**.

(a) $(4^2)^3 = (4^3)^2$ (b) $(2^3)^4 = (2^2)^6$ (c) $(3^2)^6 = (27^2)^4$

Solution:

1. (a) $(3^4)^2$
$= 3^{4(2)}$
$= 3^8$

(b) $(h^3)^{10}$
$= h^{3(10)}$
$= h^{30}$

(c) $((-y)^6)^3$
$= (-y)^{6(3)}$
$= (-y)^{18}$

2. (a) $\underbrace{(4^2)^3}_{\text{left}} = \underbrace{(4^3)^2}_{\text{right}}$

Left:
$(4^2)^3 = 4^{2(3)} = 4^6$

Same

Right:
$(4^3)^2 = 4^{3(2)} = 4^6$

Hence, $(4^2)^3 = (4^3)^2$ is **true**.

(b) $\underbrace{(2^3)^4}_{\text{left}} = \underbrace{(2^2)^6}_{\text{right}}$

Left:
$(2^3)^4 = 2^{3(4)} = 2^{12}$

Same

Right:
$(2^2)^6 = 2^{2(6)} = 2^{12}$

Hence, $(2^3)^4 = (2^2)^6$ is **true**.

(c) $\underbrace{(3^2)^6}_{\text{left}} = \underbrace{(27^2)^4}_{\text{right}}$

Left:
$(3^2)^6 = 3^{2(6)} = 3^{12}$

Right:
$(27^2)^4 = (3^{3(2)})^4$
$= 3^{6(4)}$
$= 3^{24}$

Not the same

Hence, $(3^2)^6 = (27^2)^4$ is **false**.

MIND TEST 1.2d

1. Use law of indices to simplify each of the following statements.

(a) $(12^5)^2$ (b) $(3^{10})^2$ (c) $(7^2)^3$ (d) $((-4)^3)^7$
(e) $(k^8)^3$ (f) $(g^2)^{13}$ (g) $((-m)^4)^3$ (h) $((-c)^7)^3$

2. Determine whether the following equations are **true** or **false**.

(a) $(2^4)^5 = (2^2)^{10}$ (b) $(3^3)^7 = (27^2)^4$ (c) $(5^2)^5 = (125^2)^3$ (d) $-(7^2)^4 = (-49^2)^3$

How do you use law of indices to perform operations of multiplication and division?

$(a^m \times b^n)^q$
$= (a^m)^q \times (b^n)^q$
$= a^{mq} \times b^{nq}$
→ $(a^m b^n)^q = a^{mq}\, b^{nq}$

$(a^m \div b^n)^q$
$= (a^m)^q \div (b^n)^q$
$= a^{mq} \div b^{nq}$
→ $\left(\dfrac{a^m}{b^n}\right)^q = \dfrac{a^{mq}}{b^{nq}}$

Example 10

1. Simplify each of the following.

(a) $(7^3 \times 5^4)^3$ (b) $(2^4 \times 5^3 \times 11^2)^5$ (c) $(p^2q^3r)^4$ (d) $(5m^4n^3)^2$

(e) $\left(\dfrac{2^5}{3^2}\right)^4$ (f) $\left(\dfrac{2x^3}{3y^7}\right)^4$ (g) $\dfrac{(3m^2n^3)^3}{6m^3n}$ (h) $\dfrac{(2x^3y^4)^4 \times (3xy^2)^3}{36x^{10}y^{12}}$

Solution:

(a) $(7^3 \times 5^4)^3$
$= 7^{3(3)} \times 5^{4(3)}$
$= 7^9 \times 5^{12}$

(b) $(2^4 \times 5^3 \times 11^2)^5$
$= 2^{4(5)} \times 5^{3(5)} \times 11^{2(5)}$
$= 2^{20} \times 5^{15} \times 11^{10}$

(c) $(p^2q^3r)^4$
$= p^{2(4)}\, q^{3(4)} r^{1(4)}$
$= p^8 q^{12} r^4$

(d) $(5m^4n^3)^2$
$= 5^2 m^{4(2)} n^{3(2)}$
$= 25m^8n^6$

(e) $\left(\dfrac{2^5}{3^2}\right)^4$
$= \dfrac{2^{5(4)}}{3^{2(4)}}$
$= \dfrac{2^{20}}{3^8}$

(f) $\left(\dfrac{2x^3}{3y^7}\right)^4$
$= \dfrac{2^4\, x^{3(4)}}{3^4 y^{7(4)}}$
$= \dfrac{16x^{12}}{81y^{28}}$

(g) $\dfrac{(3m^2n^3)^3}{6m^3n}$
$= \dfrac{3^3 m^{2(3)} n^{3(3)}}{6m^3n^1}$
$= \dfrac{27m^6n^9}{6m^3n^1}$
$= \dfrac{9}{2}\, m^{6-3}\, n^{9-1}$
$= \dfrac{9}{2}\, m^3\, n^8$

(h) $\dfrac{(2x^3y^4)^4 \times (3xy^2)^3}{36x^{10}y^{12}}$
$= \dfrac{2^4 x^{3(4)} y^{4(4)} \times 3^3 x^{1(3)} y^{2(3)}}{36x^{10}y^{12}}$
$= \dfrac{16x^{12}y^{16} \times 27x^3y^6}{36x^{10}y^{12}}$
$= \left(\dfrac{16 \times 27}{36}\right) x^{12+3-10} y^{16+6-12}$
$= 12x^5\, y^{10}$

FLASHBACK

$a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{mn}$

QUIZ

$m^m = 256$.
What is the value of m?

DISCUSSION CORNER

Why is $1^n = 1$ for all values of n?

Discuss.

MIND TEST 1.2e

1. Simplify each of the following.

(a) $(2 \times 3^4)^2$ (b) $(11^3 \times 9^5)^3$ (c) $(13^3 \div 7^6)^2$ (d) $(5^3 \times 3^4)^5$

(e) $(m^3n^4p^2)^5$ (f) $(2w^2x^3)^4$ (g) $\left(\dfrac{-3a^5}{b^4}\right)^6$ (h) $\left(\dfrac{2a^5}{3b^4}\right)^3$

2. Simplify each of the following.

(a) $\left(\dfrac{11^3 \times 4^2}{11^2}\right)^2$ (b) $\dfrac{3^3 \times (6^2)^3}{6^4}$ (c) $\left(\dfrac{4^2}{6^3}\right)^3 \div \dfrac{4^2}{6^3}$ (d) $\dfrac{((-4)^6)^2 \times (-5^2)^3}{(-4)^6 \times (-5)^2}$

(e) $\dfrac{x^2y^6 \times x^3}{xy^2}$ (f) $\dfrac{(h^3k^2)^4}{(hk)^2}$ (g) $\dfrac{(m^5\, n^7)^3}{(m^2n^3)^2}$ (h) $\dfrac{(b^2d^4)^3}{(b^2d^3)^2}$

3. Simplify each of the following.

(a) $\dfrac{(2m^2n^4)^3 \times (3mn^4)^2}{12m^7n^{12}}$ (b) $\dfrac{(5xy^4)^2 \times 6x^{10}y}{15x^4y^6}$ (c) $\dfrac{24d^3e^5 \times (3d^3e^4)^2}{(d^5e^6) \times (6de^2)^3}$

How do you verify $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$; $a \neq 0$?

LEARNING STANDARD

Verify that $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$; $a \neq 0$.

Brainstorming 4

In pairs

Aim: To determine the value of a number or an algebraic term with a zero index.

Steps:
1. Study and complete the following table.
2. What is your conclusion regarding zero index?

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^3$	$2^{3-3} = 2^0$	$\frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2}} = 1$	$2^0 = 1$
(d) $m^5 \div m^5$	$m^{5-5} = m^0$	$\frac{\cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m}}{\cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m} \times \cancel{m}} = 1$	$m^0 = 1$
(c) $5^4 \div 5^4$			
(d) $(-7)^2 \div (-7)^2$			
(e) $n^6 \div n^6$			

Discussion:
1. Are your answers similar with other groups?
2. What is your conclusion regarding zero index?

From Brainstorming 4, it is found that;

$$2^0 = 1$$
$$m^0 = 1$$

Therefore, a number or an algebraic term with a zero index will give a value of 1.

In general, $\boxed{a^0 = 1 \; ; a \neq 0}$

How do you verify $a^{-n} = \frac{1}{a^n}$?

Brainstorming 5

In groups

Aim: To verify $a^{-n} = \frac{1}{a^n}$.

Steps:

1. Study and complete the following table.

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^5$	$2^{3-5} = 2^{-2}$	$\frac{\cancel{2} \times \cancel{2} \times \cancel{2}}{2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}} = \frac{1}{2 \times 2} = \frac{1}{2^2}$	$2^{\boxed{-2}} = \frac{1}{2^{\boxed{2}}}$
(b) $m^2 \div m^5$	$m^{2-5} = m^{-3}$	$\frac{\cancel{m} \times \cancel{m}}{m \times m \times m \times \cancel{m} \times \cancel{m}} = \frac{1}{m \times m \times m} = \frac{1}{m^3}$	$m^{\boxed{-3}} = \frac{1}{m^{\boxed{3}}}$
(c) $3^2 \div 3^6$			
(d) $(-4)^3 \div (-4)^7$			
(e) $p^4 \div p^8$			

Discussion:

1. Are your answers similar with other groups?
2. What is your conclusion?

Scan the QR Code or visit http://bukutekskssm.my/Mathematics/F3/Chapter1AlternativeMethod.mp4 to watch a video that describes alternative method to verify $a^{-1} = \frac{1}{a^n}$.

From Brainstorming 5, it is found that;

$$2^{-2} = \frac{1}{2^2}$$

$$m^{-3} = \frac{1}{m^3}$$

In general, $\boxed{a^{-n} = \frac{1}{a^n}\ ; a \neq 0}$

BULLETIN

Negative index is a number or an algebraic term that has an index of a negative value.

Example 11

1. State each of the following terms in positive index form.

(a) a^{-2} (b) x^{-4} (c) $\frac{1}{8^{-5}}$

(d) $\frac{1}{y^{-9}}$ (e) $2m^{-3}$ (f) $\frac{3}{5}n^{-8}$

(g) $\left(\frac{2}{3}\right)^{-10}$ (h) $\left(\frac{x}{y}\right)^{-7}$

2. State each of the following in negative index form.

(a) $\frac{1}{3^4}$ (b) $\frac{1}{m^5}$ (c) 7^5

(d) n^{20} (e) $\left(\frac{4}{5}\right)^8$ (f) $\left(\frac{m}{n}\right)^{15}$

3. Simplify each of the following.

(a) $3^2 \times 3^4 \div 3^8$ (b) $\frac{(2^4)^2 \times (3^5)^3}{(2^8 \times 3^6)^2}$ (c) $\frac{(4xy^2)^2 \times x^5y}{(2x^3y)^5}$

TIPS

- $a^{-n} = \frac{1}{a^n}$
- $a^n = \frac{1}{a^{-n}}$
- $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

REMINDER

$2a^{-n} \neq \frac{1}{2a^n}$

SMART MIND

$\left(-\frac{4}{9}\right)^{-6} = x^y$

What are the values of x and y?

Solution:

1. (a) $a^{-2} = \dfrac{1}{a^2}$ (b) $x^{-4} = \dfrac{1}{x^4}$ (c) $\dfrac{1}{8^{-5}} = 8^5$ (d) $\dfrac{1}{y^{-9}} = y^9$

   (e) $2m^{-3} = \dfrac{2}{m^3}$ (f) $\dfrac{3}{5}n^{-8} = \dfrac{3}{5n^8}$ (g) $\left(\dfrac{2}{3}\right)^{-10} = \left(\dfrac{3}{2}\right)^{10}$ (h) $\left(\dfrac{x}{y}\right)^{-7} = \left(\dfrac{y}{x}\right)^7$

2. (a) $\dfrac{1}{3^4} = 3^{-4}$ (b) $\dfrac{1}{m^5} = m^{-5}$ (c) $7^5 = \dfrac{1}{7^{-5}}$ (d) $n^{20} = \dfrac{1}{n^{-20}}$

   (e) $\left(\dfrac{4}{5}\right)^8 = \left(\dfrac{5}{4}\right)^{-8}$ (f) $\left(\dfrac{m}{n}\right)^{15} = \left(\dfrac{n}{m}\right)^{-15}$

3. (a) $3^2 \times 3^4 \div 3^8$
   $= 3^{2+4-8}$
   $= 3^{-2}$
   $= \dfrac{1}{3^2}$

   (b) $\dfrac{(2^4)^2 \times (3^5)^3}{(2^8 \times 3^6)^2}$
   $= \dfrac{2^8 \times 3^{15}}{2^{16} \times 3^{12}}$
   $= 2^{8-16} \times 3^{15-12}$
   $= 2^{-8} \times 3^3$
   $= \dfrac{3^3}{2^8}$

   (c) $\dfrac{(4xy^2)^2 \times x^5y}{(2x^3y)^5}$
   $= \dfrac{4^2x^2y^4 \times x^5y^1}{2^5x^{15}y^5}$
   $= \dfrac{16}{32}x^{2+5-15}y^{4+1-5}$
   $= \dfrac{1}{2}x^{-8}y^0$
   $= \dfrac{1}{2x^8}$

TIPS

$y^0 = 1$

$y^1 = y$

MIND TEST 1.2f

1. State each of the following terms in positive index form.

   (a) 5^{-3} (b) 8^{-4} (c) x^{-8} (d) y^{-16} (e) $\dfrac{1}{a^{-4}}$

   (f) $\dfrac{1}{20^{-2}}$ (g) $3n^{-4}$ (h) $-5n^{-6}$ (i) $\dfrac{2}{7}m^{-5}$ (j) $\left(-\dfrac{3}{8}\right)^{m-4}$

   (k) $\left(\dfrac{2}{5}\right)^{-12}$ (l) $\left(-\dfrac{3}{7}\right)^{-14}$ (m) $\left(\dfrac{x}{y}\right)^{-10}$ (n) $\left(\dfrac{2x}{3y}\right)^{-4}$ (o) $\left(\dfrac{1}{2x}\right)^{-5}$

2. State each of the following terms in negative index form.

   (a) $\dfrac{1}{5^4}$ (b) $\dfrac{1}{8^3}$ (c) $\dfrac{1}{m^7}$ (d) $\dfrac{1}{n^9}$ (e) 10^2

   (f) $(-4)^3$ (g) m^{12} (h) n^{16} (i) $\left(\dfrac{4}{7}\right)^9$ (j) $\left(\dfrac{x}{\mathrm{y}}\right)^{10}$

3. Simplify each of the following.

   (a) $\dfrac{(4^2)^3 \times 4^5}{(4^6)^2}$ (b) $\dfrac{(2^3 \times 3^2)^3}{(2 \times 3^4)^5}$ (c) $\dfrac{(5^2)^5}{(2^3)^{-2} \times (5^4)^2}$

   (d) $\dfrac{3m^2n^4 \times (mn^3)^{-2}}{9m^3n^5}$ (e) $\dfrac{(2m^2n^2)^{-3} \times (3mn^2)^4}{(9m^3n)^2}$ (f) $\dfrac{(4m^2n^4)^2}{(2m^{-2}n)^5 \times (3m^4n)^2}$

How do you determine and state the relationship between fractional indices and roots and powers?

LEARNING STANDARD

Determine and state the relationship between fractional indices and roots and powers.

Relationship between $\sqrt[n]{a}$ and $a^{\frac{1}{n}}$

In Form 1, you have learnt about square and square root as well as cube and cube root. Determine the value of x for

(a) $x^2 = 9$ (b) $x^3 = 64$

TIPS

- $9 = 3^2$
- $64 = 4^3$

Solution:

(a)
$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{3^2}$$
$$x = 3$$

Square roots are used to eliminate squares.

(b)
$$x^3 = 64$$
$$\sqrt[3]{x^3} = \sqrt[3]{4^3}$$
$$x = 4$$

Cube roots are used to eliminate cubes.

Did you know that the values of x in examples (a) and (b) above can be determined by raising the index to the power of its reciprocal?

(a)
$$x^2 = 9$$
$$x^{\cancel{2}\left(\frac{1}{\cancel{2}}\right)} = 9^{\frac{1}{2}}$$
$$x^1 = 3^{\cancel{2}\left(\frac{1}{\cancel{2}}\right)}$$
$$x = 3$$

The reciprocal of 2 is $\frac{1}{2}$.

(b)
$$x^3 = 64$$
$$x^{\cancel{3}\left(\frac{1}{\cancel{3}}\right)} = 64^{\left(\frac{1}{3}\right)}$$
$$x^1 = 4^{\cancel{3}\left(\frac{1}{\cancel{3}}\right)}$$
$$x = 4$$

The reciprocal of 3 is $\frac{1}{3}$.

BULLETIN

$\frac{1}{a}$ is the reciprocal of a.

From the two methods to determine the values of x in the examples above, it is found that;

$$\sqrt[2]{x} = x^{\frac{1}{2}}$$
$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

In general, $\boxed{\sqrt[n]{a} = a^{\frac{1}{n}};\ a \neq 0}$

SMART MIND

What is the solution for $\sqrt{-4}$? Discuss.

Example 12

1. Convert each of the following terms into the form $a^{\frac{1}{n}}$.

(a) $\sqrt[2]{36}$ (b) $\sqrt[3]{-27}$ (c) $\sqrt[5]{m}$ (d) $\sqrt[7]{n}$

2. Convert each of the following terms into the form $\sqrt[n]{a}$.

(a) $125^{\frac{1}{5}}$ (b) $256^{\frac{1}{8}}$ (c) $(-1\ 000)^{\frac{1}{3}}$ (d) $n^{\frac{1}{12}}$

3. Calculate the value of each of the following terms.

(a) $\sqrt[5]{-32}$ (b) $\sqrt[6]{729}$ (c) $512^{\frac{1}{3}}$ (d) $(-243)^{\frac{1}{5}}$

Solution:

1. (a) $\sqrt[2]{36} = 36^{\frac{1}{2}}$ (b) $\sqrt[3]{-27} = (-27)^{\frac{1}{3}}$ (c) $\sqrt[5]{m} = m^{\frac{1}{5}}$ (d) $\sqrt[7]{n} = n^{\frac{1}{7}}$

2. (a) $125^{\frac{1}{5}} = \sqrt[5]{125}$ (b) $256^{\frac{1}{8}} = \sqrt[8]{256}$ (c) $(-1\ 000)^{\frac{1}{3}} = \sqrt[3]{(-1\ 000)}$ (d) $n^{\frac{1}{12}} = \sqrt[12]{n}$

3. (a) $\sqrt[5]{-32} = (-32)^{\frac{1}{5}}$
$= (-2)^{\cancel{5}\left(\frac{1}{\cancel{5}}\right)}$
$= (-2)^1$
$= -2$

(b) $\sqrt[6]{729} = 729^{\frac{1}{6}}$
$= 3^{\cancel{6}\left(\frac{1}{\cancel{6}}\right)}$
$= 3^1$
$= 3$

(c) $512^{\frac{1}{3}} = 8^{3\left(\frac{1}{3}\right)}$
$= 8^1$
$= 8$

(d) $(-243)^{\frac{1}{5}} = (-3)^{\cancel{5}\left(\frac{1}{\cancel{5}}\right)}$
$= (-3)^1$
$= -3$

TIPS

You can use a scientific calculator to check the answers.

MIND TEST 1.2g

1. Convert each of the following terms into the form $a^{\frac{1}{n}}$.
   (a) $\sqrt[3]{125}$ (b) $\sqrt[7]{2\,187}$ (c) $\sqrt[5]{-1\,024}$ (d) $\sqrt[10]{n}$
2. Convert each of the following terms into the form $\sqrt[n]{a}$.
   (a) $4^{\frac{1}{2}}$ (b) $32^{\frac{1}{5}}$ (c) $(-729)^{\frac{1}{3}}$ (d) $n^{\frac{1}{15}}$
3. Calculate the value of each of the following terms.
   (a) $\sqrt[3]{343}$ (b) $\sqrt[5]{-7\,776}$ (c) $262\,144^{\frac{1}{6}}$ (d) $(-32\,768)^{\frac{1}{5}}$

What is the relationship between $a^{\frac{m}{n}}$ and $(a^m)^{\frac{1}{n}}$, $(a^{\frac{1}{n}})^m$, $\sqrt[n]{a^m}$ dan $(\sqrt[n]{a})^m$?

You have learnt that;

$$a^{mn} = (a^m)^n \text{ and } \sqrt[n]{a^1} = a^{\frac{1}{n}}$$

From the two laws of indices above, we can convert $a^{\frac{m}{n}}$ into $(a^m)^{\frac{1}{n}}$, $(a^{\frac{1}{n}})^m$, $\sqrt[n]{a^m}$ and $(\sqrt[n]{a})^m$. Calculate the value of each of the following. Complete the table as shown in example (a).

	$a^{\frac{m}{n}}$	$(a^m)^{\frac{1}{n}}$	$(a^{\frac{1}{n}})^m$	$\sqrt[n]{a^m}$	$(\sqrt[n]{a})^m$
(a)	$64^{\frac{2}{3}}$	$(64^2)^{\frac{1}{3}}$ $= 4\,096^{\left(\frac{1}{3}\right)}$ $= 16^{3\left(\frac{1}{3}\right)}$ $= 16$	$(64^{\frac{1}{3}})^2$ $= 4^{\cancel{3}\left(\frac{1}{\cancel{3}}\right)(2)}$ $= 4^2$ $= 16$	$\sqrt[3]{64^2}$ $= \sqrt[3]{4\,096}$ $= 16$	$(\sqrt[3]{64})^2$ $= 4^2$ $= 16$
(b)	$16^{\frac{3}{4}}$				
(c)	$243^{\frac{2}{5}}$				

Are your answers in (b) and (c) the same when you use different index forms? Discuss.

From the activity above, it is found that;

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$$
$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

Example 13

1. Convert each of the following into the form $(a^m)^{\frac{1}{n}}$ and $(a^{\frac{1}{n}})^m$.
   (a) $81^{\frac{3}{2}}$ (b) $27^{\frac{2}{3}}$ (c) $h^{\frac{3}{5}}$
2. Convert each of the following into the form $\sqrt[n]{a^m}$ and $(\sqrt[n]{a})^m$.
   (a) $343^{\frac{2}{3}}$ (b) $4\,096^{\frac{5}{6}}$ (c) $m^{\frac{2}{5}}$

Solution:

1. (a) $81^{\frac{3}{2}} = (81^3)^{\frac{1}{2}}$
   $81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3$
   (b) $27^{\frac{2}{3}} = (27^2)^{\frac{1}{3}}$
   $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2$
   (c) $h^{\frac{3}{5}} = (h^3)^{\frac{1}{5}}$
   $h^{\frac{3}{5}} = (h^{\frac{1}{5}})^3$
2. (a) $343^{\frac{2}{3}} = \sqrt[3]{343^2}$
   $343^{\frac{2}{3}} = (\sqrt[3]{343})^2$
   (b) $4\,096^{\frac{5}{6}} = \sqrt[6]{4\,096^5}$
   $4\,096^{\frac{5}{6}} = (\sqrt[6]{4\,096})^5$
   (c) $m^{\frac{2}{5}} = \sqrt[5]{m^2}$
   $m^{\frac{2}{5}} = (\sqrt[5]{m})^2$

MIND TEST 1.2h

1. Complete the following table.

$a^{\frac{m}{n}}$	$729^{\frac{5}{6}}$	$121^{\frac{3}{2}}$	$w^{\frac{3}{7}}$	$x^{\frac{2}{5}}$	$\left(\frac{16}{81}\right)^{\frac{3}{4}}$	$\left(\frac{h}{k}\right)^{\frac{2}{3}}$
$(a^m)^{\frac{1}{n}}$						
$(a^{\frac{1}{n}})^m$						
$\sqrt[n]{a^m}$						
$(\sqrt[n]{a})^m$						

Example 14

1. Calculate the value of each of the following.
   (a) $9^{\frac{5}{2}}$ (b) $16^{\frac{5}{4}}$

Solution:

1. (a) $9^{\frac{5}{2}}$
   Method 1 $9^{\frac{5}{2}} = (\sqrt{9})^5 = (3)^5 = 243$
   Method 2 $9^{\frac{5}{2}} = \sqrt{9^5} = \sqrt{59\,049} = 243$
   (b) $16^{\frac{5}{4}}$
   Method 1 $16^{\frac{5}{4}} = (\sqrt[4]{16})^5 = 2^5 = 32$
   Method 2 $16^{\frac{5}{4}} = \sqrt[4]{16^5} = \sqrt[4]{1\,048\,576} = 32$

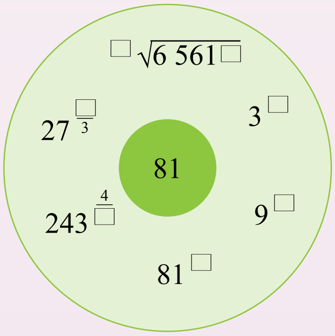
MIND TEST 1.2i

1. Calculate the value of each of the following..

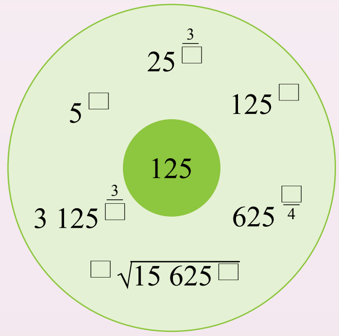
(a) $27^{\frac{2}{3}}$ (b) $32^{\frac{2}{5}}$ (c) $128^{\frac{2}{7}}$ (d) $256^{\frac{3}{8}}$
(e) $64^{\frac{4}{3}}$ (f) $1\,024^{\frac{2}{5}}$ (g) $1\,296^{\frac{3}{4}}$ (h) $49^{\frac{3}{2}}$
(i) $2\,401^{\frac{1}{4}}$ (j) $121^{\frac{3}{2}}$ (k) $2\,197^{\frac{2}{3}}$ (l) $10\,000^{\frac{3}{4}}$

2. Complete the following diagrams with correct values.

(a)



(b)



How do you perform operations involving laws of indices?

LEARNING STANDARD

Perform operations involving laws of indices.

Law of indices		
$a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$	$a^0 = 1$ $a^{-n} = \dfrac{1}{a^n}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$ $a^{\frac{m}{n}} = a^{m\left(\frac{1}{n}\right)} = \left(a^{\frac{1}{n}}\right)^m$ $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Example 15

1. Simplify each of the following.

(a) $\dfrac{(-3x)^3 \times (2x^3y^{-4})^2}{108x^4\,y^3}$ (b) $\dfrac{\sqrt{m}\,n^{\frac{3}{4}} \times (mn^3)^{\frac{1}{3}}}{(m^{-1}\,\sqrt{n^3})^{\frac{1}{6}}}$ (c) $\dfrac{(2h)^2 \times (16h^8)^{\frac{1}{4}}}{(8^{\frac{1}{3}}h)^{-2}}$

Solution:

(a)
$$\frac{(-3x)^3 \times (2x^3y^{-4})^2}{108x^4\,y^3}$$
$$= \frac{(-3)^3x^3 \times 2^2x^{3(2)}y^{-4(2)}}{108x^4\,y^3}$$
$$= \frac{-27x^3 \times 4x^6y^{-8}}{108x^4\,y^3}$$
$$= \left(\frac{-27 \times 4}{108}\right) x^{3+6-4}\,y^{-8-3}$$
$$= -1\,x^5\,y^{-11}$$
$$= -\frac{x^5}{y^{11}}$$

(b)
$$\frac{\sqrt{m}\,n^{\frac{3}{4}} \times (mn^3)^{\frac{1}{3}}}{(m^{-1}\,\sqrt{n^3})^{\frac{1}{6}}}$$
$$= \frac{m^{\frac{1}{2}}\,n^{\frac{3}{4}} \times m^{\frac{1}{3}}n^{3\left(\frac{1}{3}\right)}}{m^{-1\left(\frac{1}{6}\right)}n^{\frac{3}{2}\left(\frac{1}{6}\right)}}$$
$$= \frac{m^{\frac{1}{2}}\,n^{\frac{3}{4}} \times m^{\frac{1}{3}}\,n^1}{m^{-\frac{1}{6}}\,n^{\frac{1}{4}}}$$
$$= m^{\frac{1}{2}+\frac{1}{3}-\left(-\frac{1}{6}\right)}n^{\frac{3}{4}+1-\frac{1}{4}}$$
$$= m^1\,n^{\frac{3}{2}}$$
$$= mn^{\frac{3}{2}}$$

(c)
$$\frac{(2h)^2 \times (16h^8)^{\frac{1}{4}}}{(8^{\frac{1}{3}}h)^{-2}}$$
$$= \frac{2^2h^2 \times 16^{\frac{1}{4}}h^{8\left(\frac{1}{4}\right)}}{8^{\frac{1}{3}(-2)}h^{(-2)}}$$
$$= \frac{2^2h^2 \times 2^{\cancel{4}\left(\frac{1}{\cancel{4}}\right)}h^{\cancel{8}\left(\frac{1}{\cancel{4}}\right)}}{2^{\cancel{3}\left(\frac{1}{\cancel{3}}\right)\,(-2)}h^{(-2)}}$$
$$= \frac{2^2h^2 \times 2^1h^2}{2^{-2}\,h^{-2}}$$
$$= 2^{2+1-(-2)}\,h^{2+2-(-2)}$$
$$= 2^5\,h^6$$
$$= 32\,h^6$$

Example 16

1. Calculate the value of each of the following.

(a) $\dfrac{49^{\frac{1}{2}} \times 125^{-\frac{1}{3}}}{\sqrt[4]{2\,401} \times \sqrt[5]{3\,125}}$ (b) $\dfrac{16^{\frac{3}{4}} \times 81^{-\frac{1}{4}}}{(2^6 \times 3^4)^{\frac{1}{2}}}$ (c) $\dfrac{(243^{\frac{4}{5}} \times 5^{\frac{3}{2}})^2}{\sqrt[4]{81} \times \sqrt{25^4}}$

Solution:

(a)
$$\begin{aligned}
&\frac{49^{\frac{1}{2}} \times 125^{-\frac{1}{3}}}{\sqrt[4]{2\,401} \times \sqrt[5]{3\,125}} \\
&= \frac{7^{2(\frac{1}{2})} \times 5^{3(-\frac{1}{3})}}{(7^4)^{\frac{1}{4}} \times (5^5)^{\frac{1}{5}}} \\
&= \frac{7^1 \times 5^{-1}}{7^1 \times 5^1} \\
&= 7^{1-1} \times 5^{-1-1} \\
&= 7^0 \times 5^{-2} \\
&= 1 \times \frac{1}{5^2} \\
&= \frac{1}{25}
\end{aligned}$$

(b)
$$\begin{aligned}
&\frac{16^{\frac{3}{4}} \times 81^{-\frac{1}{4}}}{(2^6 \times 3^4)^{\frac{1}{2}}} \\
&= \frac{2^{\cancel{4}(\frac{3}{\cancel{4}})} \times 3^{4(-\frac{1}{4})}}{2^{\overset{3}{\cancel{6}}(\frac{1}{\cancel{2}})} \times 3^{\overset{2}{\cancel{4}}(\frac{1}{\cancel{2}})}} \\
&= \frac{2^3 \times 3^{-1}}{2^3 \times 3^2} \\
&= 2^{3-3} \times 3^{-1-2} \\
&= 2^0 \times 3^{-3} \\
&= 1 \times \frac{1}{3^3} \\
&= \frac{1}{27}
\end{aligned}$$

(c)
$$\begin{aligned}
&\frac{(243^{\frac{4}{5}} \times 5^{\frac{3}{2}})^2}{\sqrt[4]{81} \times \sqrt{25^4}} \\
&= \frac{243^{\frac{4}{5}(2)} \times 5^{\frac{3}{\cancel{2}}(\cancel{2})}}{81^{\frac{1}{4}} \times 25^{\frac{4}{2}}} \\
&= \frac{3^{\cancel{5}(\frac{8}{\cancel{5}})} \times 5^3}{3^{\cancel{4}(\frac{1}{\cancel{4}})} \times 5^{\cancel{2}(\frac{4}{\cancel{2}})}} \\
&= \frac{3^8 \times 5^3}{3^1 \times 5^4} \\
&= 3^{8-1} \times 5^{3-4} \\
&= 3^7 \times 5^{-1} \\
&= \frac{3^7}{5} \\
&= \frac{2\,187}{5} \\
&= 437\frac{2}{5}
\end{aligned}$$

MIND TEST 1.2j

1. Simplify each of the following.

(a) $\dfrac{\sqrt[3]{c^2d^3e} \times c^{\frac{1}{3}}d^2e^{\frac{2}{3}}}{(c^{-3}d^2e)^2}$ (b) $\dfrac{(mn^2)^3 \times (\sqrt{mn})^4}{(m^6n^3)^{\frac{2}{3}}}$ (c) $\dfrac{\sqrt{25x^3yz^2} \times 4x^2z}{\sqrt{36x^5yz^8}}$

2. Calculate the value of each of the following..

(a) $\dfrac{\sqrt{7^{-4} \times 11^4}}{49 \times 121}$ (b) $\dfrac{(5^{-3} \times 3^6)^{\frac{1}{3}} \times \sqrt[4]{16}}{(125 \times 729 \times 64)^{-\frac{1}{3}}}$ (c) $\dfrac{(2^6 \times 3^4 \times 5^2)^{\frac{3}{2}}}{\sqrt[4]{256} \times \sqrt{729} \times \sqrt[3]{125}}$

(d) $\dfrac{\sqrt[9]{512} \times \sqrt[3]{343} \times \sqrt{121}}{(64)^{\frac{1}{3}} \times (81)^{\frac{3}{4}} \times (14\,641)^{\frac{1}{4}}}$ (e) $\dfrac{(2^4 \times 3^6)^{\frac{1}{2}} \times \sqrt[3]{8} \times \sqrt{81}}{16^{\frac{3}{4}} \times 27^{\frac{1}{3}}}$ (f) $\dfrac{64^{\frac{2}{3}} \times \sqrt[3]{125} \times (2 \times \frac{1}{5})^{-3}}{4^2 \times \sqrt[4]{625}}$

3. Given $m = 2$ and $n = -3$, calculate the value of $64^{\frac{m}{3}} \times 512^{(-\frac{1}{n})} \div 81^{\frac{n}{2m}}$.

4. Given $a = \dfrac{1}{2}$ and $b = \dfrac{2}{3}$, calculate the value of $144^a \div 64^b \times 256^{-\frac{a}{b}}$.

How do you solve problems involving laws of indices?

LEARNING STANDARD

Solve problems involving laws of indices.

FLASHBACK

Common prime factors of 6 and 12 are 2 and 3.

Example 17

Calculate the value of $\sqrt{3} \times 12^{\frac{3}{2}} \div 6$ without using a calculator.

Understanding the problem

Calculate the value of numbers given in index form with different bases.

Planning a strategy

Convert each base into prime factors and calculate the value by applying laws of indices.

Implementing the strategy

$$\begin{aligned}
&\sqrt{3} \times 12^{\frac{3}{2}} \div 6 \\
&= 3^{\frac{1}{2}} \times (2 \times 2 \times 3)^{\frac{3}{2}} \div (2 \times 3) \\
&= 3^{\frac{1}{2}} \times 2^{\frac{3}{2}} \times 2^{\frac{3}{2}} \times 3^{\frac{3}{2}} \div (2^1 \times 3^1) \\
&= 3^{\frac{1}{2} + \frac{3}{2} - 1} \times 2^{\frac{3}{2} + \frac{3}{2} - 1} \\
&= 3^1 \times 2^2 \\
&= 12
\end{aligned}$$

Making a conclusion

$\sqrt{3} \times 12^{\frac{3}{2}} \div 6 = 12$

Example 18

Calculate the value of x for the equation $3^x \times 9^{x+5} \div 3^4 = 1$.

REMINDER

- If $a^m = a^n$ then, $m = n$
- If $a^m = b^m$ then, $a = b$

Understanding the problem

Calculate the value of variable x which is part of the indices indeks.

Planning a strategy

The question is an equation. Hence, the value on the left side of the equation is the same as the value on the right side of the equation. Convert all the terms into index form with base of 3.

Implementing the strategy

$$\begin{aligned}
3^x \times 9^{x+5} \div 3^4 &= 1 \\
3^x \times 3^{2(x+5)} \div 3^4 &= 3^0 \\
3^{x+2(x+5)-4} &= 3^0 \\
3^{x+2x+10-4} &= 3^0 \\
3^{3x+6} &= 3^0
\end{aligned}$$

$a^m = a^n$
$m = n$

$$\begin{aligned}
3x + 6 &= 0 \\
3x &= -6 \\
x &= \frac{-6}{3} \\
x &= -2
\end{aligned}$$

Making a conclusion

If $3^x \times 9^{x+5} \div 3^4 = 1$, then, $x = -2$

Checking Answers

You can check the answer by substituting the value of x into the original equation.

$\underbrace{3^x \times 9^{x+5} \div 3^4}_{\text{Left}} = \underbrace{1}_{\text{Right}}$

Substitute $x = -2$ into left side of the equation

$$\begin{aligned}
&3^{-2} \times 9^{-2+5} \div 3^4 \\
&= 3^{-2} \times 9^3 \div 3^4 \\
&= 3^{-2} \times 3^{2(3)} \div 3^4 \\
&= 3^{-2+6-4} \\
&= 3^0 \\
&= 1
\end{aligned}$$

The same value as the value on the right side of the equation.

Example 19

Calculate the possible values of x for the equation $3^{x^2} \times 3^{2x} = 3^{15}$.

Understanding the problem

Calculate the value of x which is part of the indices.

Planning a strategy

All the bases involved in the equation are the same.

Implementing the strategy

$3^{x^2} \times 3^{2x} = 3^{15}$

$3^{x^2 + 2x} = 3^{15}$

$x^2 + 2x = 15$

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x - 3 = 0$ or $x + 5 = 0$

$x = 0 + 3$ $\quad$ $x = 0 - 5$

$x = 3$ $\quad$ $x = -5$

If $a^m = a^n$, then, $m = n$.

Solve the quadratic equation using factorisation method.

Making a conclusion

The possible values of x for the equation $3^{x^2} \times 3^{2x} = 3^{15}$ are 3 and –5.

Checking Answers

Substitute the values of x into the original equation.

$\underbrace{3^{x^2} \times 3^{2x}}_{\text{Left}} = \underbrace{3^{15}}_{\text{Right}}$

Substitute $x = 3$

Left:

$3^{(3)^2} \times 3^{2(3)}$

$= 3^9 \times 3^6$

$= 3^{9+6}$

$= 3^{15}$

Right:

3^{15}

The same value

Substitute $x = -5$

Left:

$3^{(-5)^2} \times 3^{2(-5)}$

$= 3^{25} \times 3^{-10}$

$= 3^{25 + (-10)}$

$= 3^{15}$

Right:

3^{15}

The same value

Example 20

Solve the following simultaneous equations.

$$25^m \times 5^n = 5^8 \text{ and } 2^m \times \frac{1}{2^n} = 2$$

Solution:

$25^m \times 5^n = 5^8$

$5^{2(m)} \times 5^n = 5^8$

$5^{2m+n} = 5^8$

$2m + n = 8 \rightarrow$ (1)

$2^m \times \frac{1}{2^n} = 2$

$2^m \times 2^{-n} = 2^1$

$2^{m+(-n)} = 2^1$

$m - n = 1 \rightarrow$ (2)

Equation (1) and (2) can be solved by substitution method.

From (1):

$2m + n = 8$

$n = 8 - 2m \rightarrow$ (3)

Substitute (3) into (2)

$m - n = 1$

$m - (8 - 2m) = 1$

$m - 8 + 2m = 1$

$m + 2m = 1 + 8$

$3m = 9$

$m = \frac{9}{3}$

$m = 3$

Substitute $m = 3$ into (1)

$2m + n = 8$

$2(3) + n = 8$

$6 + n = 8$

$n = 8 - 6$

$n = 2$

You can also substitute $m = 3$ into equation (2) or (3).

Hence, $m = 3$ and $n = 2$.

FLASHBACK

Simultaneous linear equations in two variables can be solved using substitution method or elimination method.

Checking Answers

Substitute $m = 3$ and $n = 2$ into original simultaneous equations.

$\underbrace{25^m \times 5^n}_{\text{Left}} = \underbrace{5^8}_{\text{Right}}$

Left:

$25^m \times 5^n$

$= 5^{2(m)} \times 5^n$

$= 5^{2(3)} \times 5^2$

$= 5^{6+2}$

$= 5^8$

Right:

5^8

The same value

$\underbrace{2^m \times \frac{1}{2^n}}_{\text{Left}} = \underbrace{2}_{\text{Right}}$

Left:

$2^m \times \frac{1}{2^n}$

$= 2^3 \times \frac{1}{2^2}$

$= 2^3 \times 2^{-2}$

$= 2^{3+(-2)}$

$= 2^1$

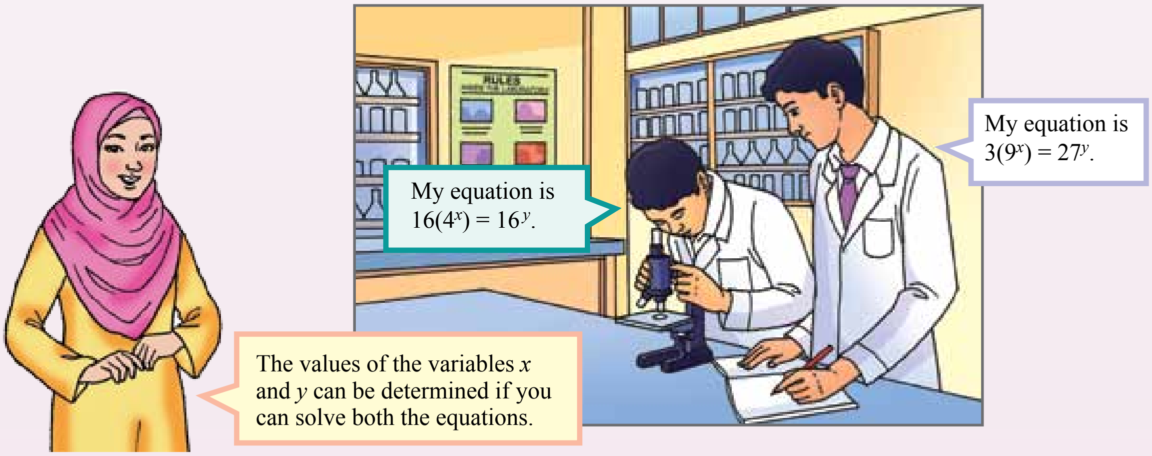
$= 2$

Right:

2

The same value

Example 21



Chong and Navin performed an experiment to determine the relationship between variable x and variable y. The equation Chong obtained was $16(4^x) = 16^y$, while the equation Navin got was $3(9^x) = 27^y$ as the findings of the experiment they performed. Calculate the value of x and of y which satisfy both the experiments Chong and Navin have performed.

Solution:

$$16(4^x) = 16^y \qquad 3(9^x) = 27^y$$
$$4^2(4^x) = 4^{2(y)} \qquad 3(3^{2x}) = 3^{3(y)}$$
$$4^{2+x} = 4^{2y} \qquad 3^{1+2x} = 3^{3y}$$
$$2 + x = 2y \rightarrow (1) \qquad 1 + 2x = 3y \rightarrow (2)$$

You can also substitute $y = 3$ into equation (2) or (3).

Equations (1) and (2) can be solved by elimination method.

Multiply equation (1) by 2 to equate the coefficients of variable x.

$(1) \times 2 : 4 + 2x = 4y \rightarrow (3)$
$(2) \quad : 1 + 2x = 3y$
$(3) - (2)$:
$3 + 0 = y$
$y = 3$

Substitute $y = 3$ into equation (1)

$(1) : 2 + x = 2y$
$2 + x = 2(3)$
$x = 6 - 2$
$x = 4$

Hence, $x = 4$, $y = 3$

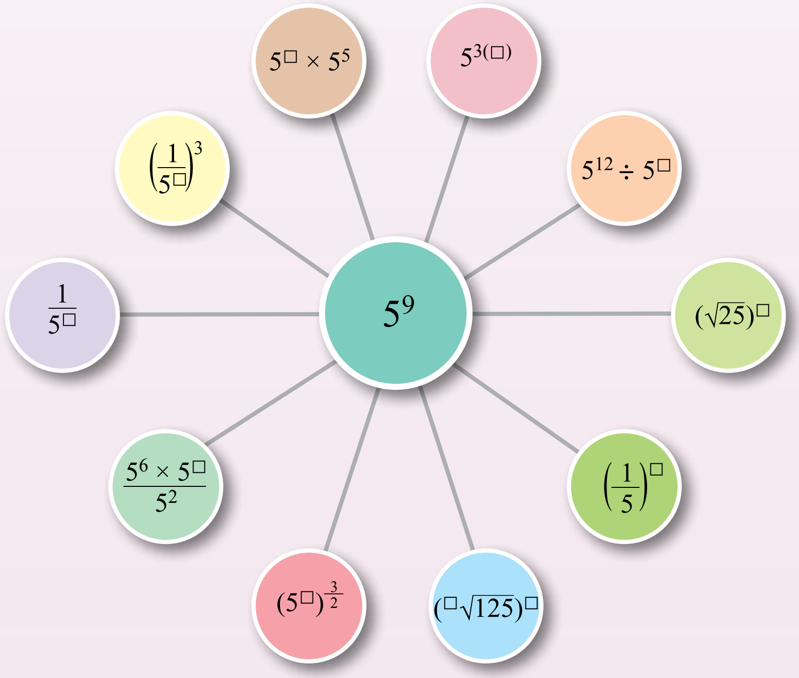
Dynamic Challenge

Test Yourself

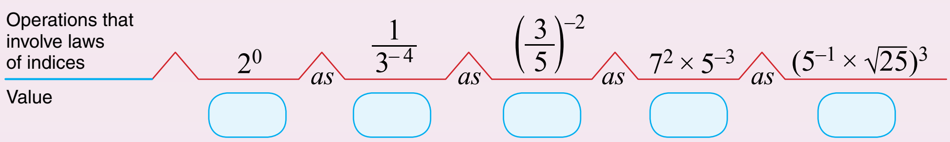
1. State whether each of the following operations which involves the laws of indices is **true** or **false**. If it is false, state the correct answer.

(a) $a^5 = a \times a \times a \times a \times a$
(b) $5^2 = 10$
(c) $3^0 = 0$
(d) $(2x^3)^5 = 2x^{15}$
(e) $m^0n^0 = 1$
(f) $2a^{-4} = \dfrac{1}{2a^4}$
(g) $32^{\frac{2}{5}} = (\sqrt[2]{32})^5$
(h) $\left(\dfrac{m}{n}\right)^{-4} = \left(\dfrac{n}{m}\right)^4$
(i) $(5m^{\frac{1}{4}})^{-4} = \dfrac{625}{m}$

2. Copy and complete the following diagram with suitable values.



3. Copy and complete the following diagram.



Skills Enhancement

1. Simplify each of the following.

(a) $(mn^4)^3 \div m^4n^5$ (b) $3x \times \frac{1}{6}y^4 \times (xy)^3$ (c) $\sqrt{xy} \times \sqrt[3]{xy^2} \times \sqrt[6]{xy^5}$

2. Calculate the value of each of the following.

(a) $64^{\frac{1}{3}} \times 5^{-3}$ (b) $7^{-1} \times 125^{\frac{2}{3}}$ (c) $(256)^{\frac{3}{8}} \times 2^{-3}$

(d) $2^4 \times 16^{-\frac{3}{4}}$ (e) $\sqrt{49} \times 3^{-2} \div (\sqrt{81})^{-1}$ (f) $(125)^{\frac{2}{3}} \times (25)^{-\frac{3}{2}} \div (625)^{-\frac{1}{4}}$

3. Calculate the value of x for each of the following equations.

(a) $2^6 \div 2^x = 8$ (b) $3^{-4} \times 81 = 3^x$ (c) $a^xa^8 = 1$

(d) $4 \times 8^{x+1} = 2^{2x}$ (e) $(a^x)^2 \times a^5 = a^{3x}$ (f) $2^x = \frac{2^{10}}{16^x}$

(g) $3^6 \div 3^x = 81^{(x-1)}$ (h) $(m^2)^x \times m^{(x+1)} = m^{-2}$ (i) $25^x \div 125 = \frac{1}{5^x}$

Self Mastery

1. Calculate the value of each of the following without using a calculator.

(a) $4^{\frac{1}{3}} \times 50^{\frac{2}{3}} \times 10^{\frac{5}{3}}$ (b) $5^{\frac{5}{2}} \times 20^{\frac{3}{2}} \div 10^{-2}$ (c) $60^{\frac{1}{2}} \times 125^{\frac{2}{3}} \div \sqrt{15}$

2. Calculate the value of x for each of the following equations.

(a) $64x^{\frac{1}{2}} = 27x^{-\frac{5}{2}}$ (b) $3x^{\frac{2}{3}} = \frac{27}{4}x^{-\frac{4}{3}}$ (c) $25x^{-\frac{2}{3}} - \frac{5}{3}x^{\frac{1}{3}} = 0$

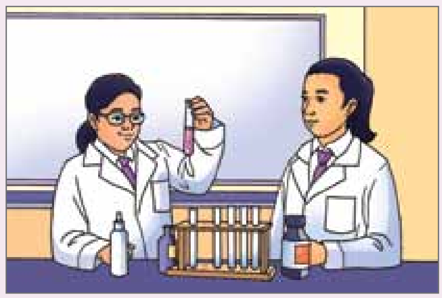
3. Calculate the possible values of x for each of the following equations.

(a) $a^{x^2} \div a^{5x} = a^6$ (b) $2^{x^2} \times 2^{6x} = 2^7$ (c) $5^{x^2} \div 5^{3x} = 625$

4. Solve the following simultaneous equations.

(a) $81^{(x+1)} \times 9^x = 3^5$ and $8^{2x} \times 4(2^{2y}) = 128$ (b) $4(4^x) = 8^{y+2}$ and $9^x \times 27^y = 1$

5. In an experiment performed by Susan, it was found that the temperature of a metal rose from 25°C to T°C according to equation $T = 25(1.2)^m$ when the metal was heated for m seconds. Calculate the difference in temperature between the fifth second and the sixth second, to the nearest degree Celsius.



6. Encik Azmi bought a locally made car for RM55 000. After 6 years, Encik Azmi wishes to sell the car. Based on the explanation from the used car buyers, the price of Encik Azmi's car will be calculated by the formula RM55 000 $\left(\frac{8}{9}\right)^n$. In this situation, n is the number of years after the car is bought. What is the market value of Encik Azmi's car? State your answer correct to the nearest RM.

7. Mrs Kiran Kaur saved RM50 000 on 1 March 2019 in a local bank with an interest of 3.5% per annum. After t years, Mrs Kiran Kaur's total savings, in RM, is 50 000 $(1.035)^t$. Calculate her total savings on 1 March 2025, if Mrs Kiran Kaur does not withdraw her savings.

PROJECT

Materials: One sheet of A4 paper, a pair of scissors, a long ruler, a pencil.

Instructions: (a) Carry out the project in small groups.
(b) Cut the A4 paper into the shape of a square. (Biggest possible)

Steps:

1. Draw the axes of symmetry (vertical and horizontal only) as shown in Diagram 1.
2. Calculate the number of squares formed. Write your answers in the space provided in Sheet A.
3. Draw the vertical and horizontal axes of symmetry for each square as shown in Diagram 2.
4. Calculate the number of squares formed. Write your answers in the space provided in Sheet A.
5. Repeat step 3 and step 4 as many times as possible.

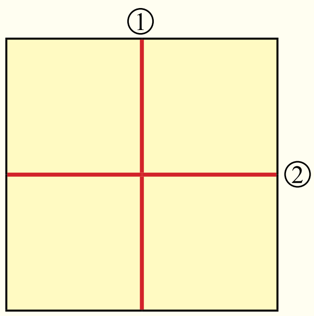


Diagram 1

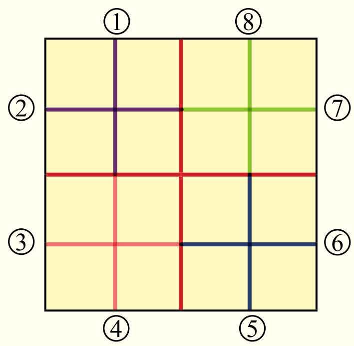


Diagram 2

6. Compare your answers with those of other groups.
7. What can you say about the patterns in the column 'Index form' in Sheet A?
8. Discuss the patterns you identify.

Scan the QR Code or visit http://bukuteksksm.my/Mathematics/F3/Chapter1SheetA.pdf to download Sheet A.

Sheet A

Number of axes of symmetry	Index form	Number of squares	Index form
0	–	1	2^0
2	2^1	4	2^2
8		16	

CONCEPT MAP

Indices

a^n ← Index; a ← Base	$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}}$	$5^4 = 5 \times 5 \times 5 \times 5$ $m \times m \times m \times m \times m = m^5$

Multiplication

$a^m \times a^n = a^{m+n}$

$2^3 \times 2^5 = 2^{3+5}$

Division

$a^m \div a^n = a^{m-n}$

$3^6 \div 3^4 = 3^{6-4}$

Power

$(a^m)^n = a^{mn}$	$(a^m \times a^n)^p = a^{mp} \times a^{np}$
$(3^4)^2 = 3^8$	$(3a^4)^3 = 27a^{12}$

Fractional index

$a^{\frac{1}{n}} = \sqrt[n]{a}$	$8^{\frac{1}{3}} = \sqrt[3]{8}$
$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$	$8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = (8^{\frac{1}{3}})^2$
$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2$

Negative index

$a^{-n} = \frac{1}{a^n}$; $a \neq 0$

$5^{-3} = \frac{1}{5^3}$

Zero index

$a^0 = 1$; $a \neq 0$

$2^0 = 1$

$m^0 = 1$

SELF-REFLECT

At the end of this chapter, I can:

		🙂	☹️
1.	Represent repeated multiplication in index form and describe its meaning.		
2.	Rewrite a number in index form and vice versa.		
3.	Relate the multiplication of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.		
4.	Relate the division of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.		
5.	Relate the numbers in index form raised to a power, to repeated multiplication, and hence make generalisation.		
6.	Verify that $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$; $a \neq 0$.		
7.	Determine and state the relationship between fractional indices and roots and powers.		
8.	Perform operations involving laws of indices.		
9.	Solve problems involving laws of indices.		

EXPLORING MATHEMATICS

Do you still remember the Pascal's Triangle that you learnt in the Chapter 1 Patterns and Sequences in Form 2?

The Pascal's Triangle, invented by a French mathematician, Blaise Pascal, has a lot of unique properties. Let us explore two unique properties found in the Pascal's Triangle.

Activity 1

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Sheet 1

Sum	Index form
1	2^0
2	2^1
4	2^2

Sheet 1(a)

Instructions:

1. Carry out the activity in pairs.
2. Construct the Pascal's Triangle as in Sheet 1.
3. Calculate the sum of the numbers in each row. Write the sum in index form with base of 2.
4. Complete Sheet 1(a). Discuss with your friends about the patterns of answers obtained.
5. Present your results.

TIPS

$11^5 = 161\ 051$

1	5	10	10	5	1
	+1	+1			
1	6	1	0	5	1

Activity 2

11^n	Value
11^0	1
11^1	11
11^2	121
11^3	1 331
11^4	
11^5	
11^6	
11^7	
11^8	
11^9	
11^{10}	

Sheet 2(a)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Sheet 2

Instructions:

1. Carry out the activity in small groups.
2. Construct the Pascal's Triangle as in Sheet 2.
3. Take note on the numbers in each row. Each number is the value of index with base of 11.
4. Complete Sheet 2(a) with the value of index with base of 11 without using a calculator.
5. Present your results.
6. Are your answers the same as those of other groups?