KURIKULUM STANDARD SEKOLAH MENENGAH

MATHEMATICS
FORM 3

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Contents

iii

Introduction

This Form 3 Mathematics Textbook is prepared based on *Kurikulum Standard Sekolah Menengah (KSSM)*. This book contains 9 chapters arranged systematically based on Form 3 Mathematics *Dokumen Standard Kurikulum dan Pentaksiran (DSKP)*.

At the beginning of each chapter, students are introduced to stimulating materials related to daily life to stimulate their thinking about the topic. In addition, Learning Standard and word list also give a visual summary about the chapter's content.

This book contains the following special features:

	Description
What will you learn?	Contains learning standard that students will learn in each chapter.
Why do you learn this chapter?	Applications of knowledge in this chapter in related career fields.
Exploring Era	History of ancient academy or original exploration of the chapter in Mathematics.
WORD BANK	Word list contained in each chapter.
Brainstorming Individual In pairs In groups	Helps students to understand the basic mathematical concept via individual, pair or group activities.
BULLETIN	Gives additional information about the chapter learned.
QUIZ	Questions that test students' capability to understand certain technique in each chapter.
REMINDER	Grabs students' attention to additional facts that need to be reminded of, mistakes that students commonly make, and carelessness to be avoided.
TIPS	Exposes students to additional knowledge that they need to know.
SMART MIND	Presents mind-stimulating questions for enhancement of students' critical and creative thinking.

		Description
SMART TECHNOLOGY	⟩⟩⟩	Exposes students to the use of technological tools in the learning of mathematics.
DISCUSSION CORNER	⟩⟩⟩	Develops communication skills mathematically.
FLASHBACK	⟩⟩⟩	Helps students to remember what they have learnt.
SMART FINGER	⟩⟩⟩	Shows the use of scientific calculators in calculations.
PROJECT	⟩⟩⟩	Enables students to carry out assignments and then present their completed work in class.
MIND TEST	⟩⟩⟩	Test students' understanding on the concepts they have learnt.
	⟩⟩⟩	Indicates HOTS questions to help in developing students' higher order thinking skills.
Dynamic Challenge	⟩⟩⟩	Prepares more diversified exercises which incorporate the elements of LOTS, HOTS, TIMSS and PISA assessment.
	⟩⟩⟩	Enables students to scan QR Code using mobile device.
EXPLORING MATHEMATICS	⟩⟩⟩	Covers applicable concepts of digital tool calculators, hands on activities and games that aim to provides additional activities to effectively enhance students' understanding.
CONCEPT MAP	⟩⟩⟩	Overall chapter summary that students learnt.
SELF-REFLECT	⟩⟩⟩	Looks back whether students have achieved the learning standard.
Checking Answers	⟩⟩⟩	Checks answers with alternative methods.
STEM	⟩⟩⟩	Activities with elements of Science, Technology, Engineering and Mathematics.
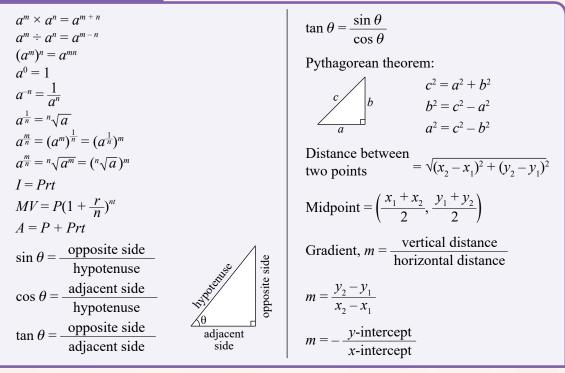
Symbols and Formulae

SYMBOLS

$\sqrt{}$	root	$\geqslant$	is more than or equal to
π	pi	$<$	is less than
$a : b$	ratio of a to b	$\leqslant$	is less than or equal to
$A \times 10^n$	standard form where	Δ	triangle
	$1 \leqslant A < 10$ and n is an integer	$\angle$	angle
$=$	is equal to	$^\circ$	degree
$\approx$	is approximately equal to	$'$	minute
$\neq$	is not equal to	$''$	second
$>$	is more than		

FORMULAE

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

$a^{-n} = \dfrac{1}{a^n}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$I = Prt$

$MV = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$A = P + Prt$

$\sin \theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos \theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan \theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

$\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

Pythagorean theorem:

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$a^2 = c^2 - b^2$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\text{Midpoint} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$\text{Gradient, } m = \dfrac{\text{vertical distance}}{\text{horizontal distance}}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = -\dfrac{y\text{-intercept}}{x\text{-intercept}}$
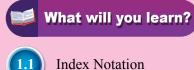
Download the free *QR Code* scanner to your mobile devices. Scan *QR Code* or visit the website http://bukutekskssm.my/Mathematics/F3/Index.html to download files for brainstorming. Then, save the downloaded file for offline use.

Note: Students can download free *GeoGebra and Geometer's Sketchpad* (*GSP*) software to open related files.

http://bukutekskssm.my/Mathematics/F3/Index.html

KPM

CHAPTER 1

Indices

What will you learn?

1.1 Index Notation
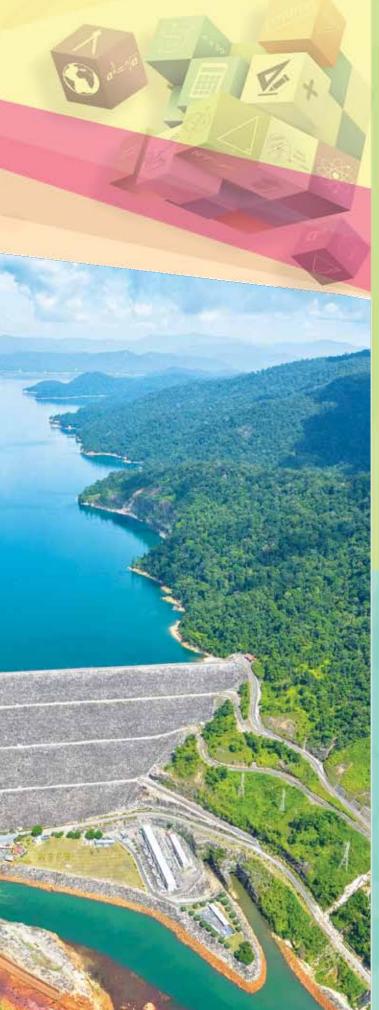
1.2 Law of Indices

Why do you learn this chapter?

- Writing a number in index notation enables the number stated in a simple and easily understood form. Various operations of mathematics that involve numbers in index notation can be performed by using laws of indices.

- Concept of index is used in the fields of science, engineering, accounting, finance, astronomy, computer and so on.

Kenyir Lake, located in the district of Hulu Terengganu, in Terengganu, is the biggest man-made lake in Southeast Asia. Kenyir Lake is a world famous tourist destination known for its unique natural beauty. Kenyir Lake is an important water catchment area. Kenyir Lake, which was built in the year 1985, supplies water to Sultan Mahmud Power Station. The estimated catchment area at the main dam is 2 600 km^2 with a reservoir volume of 13 600 million cubic metre. During rainy season, the volume of water in the catchment area will increase sharply. What action should be taken to address this situation?

Index notation is an important element in the development of mathematics and computer programming. The use of positive indices was introduced by Rene Descartes (1637), a well-known French mathematician. Sir Isaac Newton, another well-known British mathematician, developed the field of index notation and introduced negative indices and fractional indices.

WORD BANK

• base	• *asas*
• factor	• *faktor*
• index	• *indeks*
• fractional index	• *indeks pecahan*
• power	• *kuasa*
• root	• *punca kuasa*
• index notation	• *tatatanda indeks*

1.1 Index Notation

✎ What is repeated multiplication in index form?

The development of technology not only makes most of our daily tasks easier, it also saves cost of expenses in various fields. For instance, the use of memory cards in digital camera enables users to store photographs in a large number and to delete or edit unsuitable photographs before printing.

LEARNING STANDARD

Represent repeated multiplication in index form and describe its meaning.

DISCUSSION CORNER 💬

Discuss the value of the capacity of a pen drive.

BULLETIN 📢

The nuclear fission of uranium U-320 follows the pattern 30, 31, 32, …

In the early stage, memory cards were made with a capacity of 4MB. The capacity was increased with time and the needs of users. Did you know that the value of capacity of memory cards is calculated using a special form that is 2^n?

In Form 1, you have learnt that $4^3 = 4 \times 4 \times 4$. The number 4^3 is written in index notation, 4 is the **base** and 3 is the **index** or **exponent**. The number is read as '4 to the power of 3'.

Hence, a number in index notation or in index form can be written as;

$$a^n \leftarrow \text{Index}$$
$$\uparrow \text{ Base}$$

You have also learnt that $4^2 = 4 \times 4$ and $4^3 = 4 \times 4 \times 4$. For example;

$4 \times 4 = 4^{②}$ ⟶ The value of index is 2

Repeated two times ⟶ The value of index is the same as the number of times 4 is multiplied repeatedly.

$4 \times 4 \times 4 = 4^{③}$ ⟶ The value of index is 3

Repeated three times ⟶ The value of index is the same as the number of times 4 is multiplied repeatedly.

Example 1

Write the following repeated multiplications in index form a^n.

(a) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

(b) $0.3 \times 0.3 \times 0.3 \times 0.3$

(c) $(-2) \times (-2) \times (-2)$

(d) $\dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4}$

(e) $m \times m \times m \times m \times m \times m \times m$

(f) $n \times n \times n \times n \times n \times n \times n \times n$

REMINDER ✏️

$2^5 \neq 2 \times 5$ $4^3 \neq 4 \times 3$

$a^n \neq a \times n$

Solution:

(a) $\underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{repeated six times}} = 5^6$

(b) $\underbrace{0.3 \times 0.3 \times 0.3 \times 0.3}_{\text{repeated four times}} = (0.3)^4$

(c) $\underbrace{(-2) \times (-2) \times (-2)}_{\text{repeated three times}} = (-2)^3$

(d) $\underbrace{\dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4} \times \dfrac{1}{4}}_{\text{repeated five times}} = \left(\dfrac{1}{4}\right)^5$

(e) $\underbrace{m \times m \times m \times m \times m \times m \times m}_{\text{repeated seven times}} = m^7$
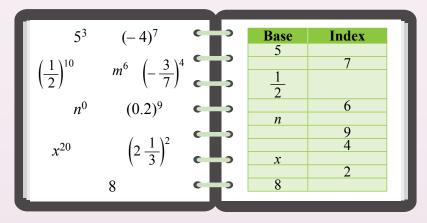
(f) $\underbrace{n \times n \times n \times n \times n \times n \times n \times n}_{\text{repeated eight times}} = n^8$

From the solution in Example 1, it is found that the value of index in an index form is the same as the number of times the base is multiplied repeatedly. In general,

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}} \; ; \; a \neq 0$$

MIND TEST **1.1a**

1. Complete the following table with base or index for the given numbers or algebraic terms.

$5^3 \qquad (-4)^7$

$\left(\dfrac{1}{2}\right)^{10} \qquad m^6 \qquad \left(-\dfrac{3}{7}\right)^4$

$n^0 \qquad (0.2)^9$

$x^{20} \qquad \left(2\dfrac{1}{3}\right)^2$

8

Base	Index
5	
	7
$\dfrac{1}{2}$	
	6
n	
	9
	4
x	
	2
8	

2. State the following repeated multiplications in index form a^n.

    (a) $6 \times 6 \times 6 \times 6 \times 6 \times 6$

    (b) $0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$

    (c) $\dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2}$

    (d) $(-m) \times (-m) \times (-m) \times (-m) \times (-m)$

    (e) $1\dfrac{2}{3} \times 1\dfrac{2}{3} \times 1\dfrac{2}{3}$

    (f) $\left(-\dfrac{1}{n}\right) \times \left(-\dfrac{1}{n}\right) \times \left(-\dfrac{1}{n}\right) \times \left(-\dfrac{1}{n}\right) \times \left(-\dfrac{1}{n}\right) \times \left(-\dfrac{1}{n}\right)$

3. Convert the numbers or algebraic terms in index form into repeated multiplications.

    (a) $(-3)^3$

    (b) $(2.5)^4$

    (c) $\left(\dfrac{2}{3}\right)^5$

    (d) $\left(-2\dfrac{1}{4}\right)^3$

    (e) k^6

    (f) $(-p)^7$

    (g) $\left(\dfrac{1}{m}\right)^8$

    (h) $(3n)^5$

How do you convert a number into a number in index form?

A number can be written in index form if a suitable base is selected. You can use repeated division method or repeated multiplication method to convert a number into a number in index form.

Example 2
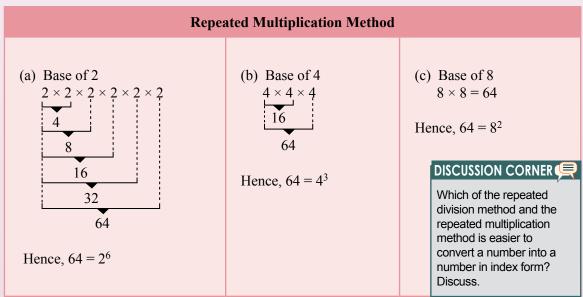
Write 64 in index form using base of 2, base of 4 and base of 8.

FLASHBACK

$4 \times 4 \times 4 = 4^3$

Solution:

Repeated Division Method		
(a) Base of 2 • 64 is divided repeatedly by 2. $n = 6\begin{cases}2) 64 \\ 2) 32 \\ 2) 16 \\ 2) \underline{\ 8} \\ 2) \underline{\ 4} \\ 2) \underline{\ 2} \\ \qquad 1\end{cases}$ Hence, $64 = 2^6$	(b) Base of 4 • 64 is divided repeatedly by 4. $n = 3\begin{cases}4) 64 \\ 4) 16 \\ 4) \underline{\ 4} \\ \quad 1\end{cases}$ Hence, $64 = 4^3$	(c) Base of 8 • 64 is divided repeatedly by 8. $n = 2\begin{cases}8) 64 \\ 8) \underline{\ 8} \\ \quad 1\end{cases}$ Hence, $64 = 8^2$

The division is continued until 1 is obtained.

Repeated Multiplication Method		
(a) Base of 2 $2 \times 2 \times 2 \times 2 \times 2 \times 2$ $\quad 4$ $\qquad 8$ $\qquad\ 16$ $\qquad\quad 32$ $\qquad\qquad 64$ Hence, $64 = 2^6$	(b) Base of 4 $4 \times 4 \times 4$ $\quad 16$ $\qquad 64$ Hence, $64 = 4^3$	(c) Base of 8 $8 \times 8 = 64$ Hence, $64 = 8^2$

DISCUSSION CORNER

Which of the repeated division method and the repeated multiplication method is easier to convert a number into a number in index form? Discuss.

Example 3

Write $\dfrac{32}{3\,125}$ in index form using base of $\dfrac{2}{5}$.

Solution:

Repeated Division Method

$n = 5$
$$\begin{array}{r} 2\,)\,32 \\ 2\,)\,16 \\ 2\,)\,\ \ 8 \\ 2\,)\,\ \ 4 \\ 2\,)\,\ \ 2 \\ 1 \end{array}$$

$n = 5$
$$\begin{array}{r} 5\,)\,3\,125 \\ 5\,)\,\ \ \ 625 \\ 5\,)\,\ \ \ 125 \\ 5\,)\,\ \ \ \ 25 \\ 5\,)\,\ \ \ \ \ 5 \\ 1 \end{array}$$

Hence, $\dfrac{32}{3\,125} = \left(\dfrac{2}{5}\right)^5$

Repeated Multiplication Method

$$\dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5} \times \dfrac{2}{5}$$

$$\dfrac{4}{25}$$

$$\dfrac{8}{125}$$

$$\dfrac{16}{625}$$

$$\dfrac{32}{3\,125}$$

Hence, $\dfrac{32}{3\,125} = \left(\dfrac{2}{5}\right)^5$

MIND TEST 1.1b

1. Write each of the following numbers in index form using the stated base in brackets.

   (a) 81 [base of 3]   (b) 15 625 [base of 5]   (c) $\dfrac{64}{125}$ $\left[\text{base of } \dfrac{4}{5}\right]$

   (d) 0.00032 [base of 0.2]   (e) − 16 384 [base of (− 4)]   (f) $\dfrac{1}{16}$ $\left[\text{base of } \left(-\dfrac{1}{4}\right)\right]$

How do you determine the value of the number in index form , a^n?

The value of a^n can be determined by repeated multiplication method or using a scientific calculator.

Example 4

Calculate the values of the given numbers in index form.

(a) 2^5

$$2 \times 2 \times 2 \times 2 \times 2$$
$$4 \ \times \ 2$$
$$8 \ \times \ 2$$
$$16 \ \times \ 2$$
$$32$$

Hence, $2^5 = 32$

(b) $(0.6)^3$

$$0.6 \times 0.6 \times 0.6$$
$$0.36 \ \times \ 0.6$$
$$0.216$$

$0.6^3 = 0.216$
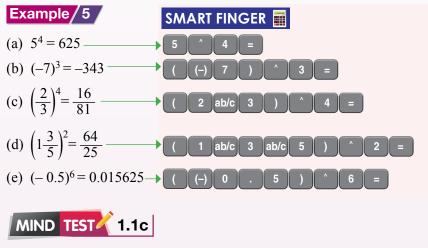
Hence, $0.6^3 = 0.216$

QUIZ

$(m)^4 = 16$
What are the possible values of m?

Example 5

(a) $5^4 = 625$ →

$$\boxed{5}\ \boxed{\wedge}\ \boxed{4}\ \boxed{=}$$

(b) $(-7)^3 = -343$ →

$$\boxed{(}\ \boxed{(-)}\ \boxed{7}\ \boxed{)}\ \boxed{\wedge}\ \boxed{3}\ \boxed{=}$$

(c) $\left(\dfrac{2}{3}\right)^4 = \dfrac{16}{81}$ →

$$\boxed{(}\ \boxed{2}\ \boxed{ab/c}\ \boxed{3}\ \boxed{)}\ \boxed{\wedge}\ \boxed{4}\ \boxed{=}$$

(d) $\left(1\dfrac{3}{5}\right)^2 = \dfrac{64}{25}$ →

$$\boxed{(}\ \boxed{1}\ \boxed{ab/c}\ \boxed{3}\ \boxed{ab/c}\ \boxed{5}\ \boxed{)}\ \boxed{\wedge}\ \boxed{2}\ \boxed{=}$$

(e) $(-0.5)^6 = 0.015625$ →

$$\boxed{(}\ \boxed{(-)}\ \boxed{0}\ \boxed{.}\ \boxed{5}\ \boxed{)}\ \boxed{\wedge}\ \boxed{6}\ \boxed{=}$$

SMART FINGER 🖩

REMINDER ✏
Negative or fractional base must be placed within brackets when using a calculator to calculate values of given numbers.

DISCUSSION CORNER 💬
Calculate questions (c), (d) and (e) in Example 5 without using brackets. Are the answers the same? Discuss.

MIND TEST ✏ 1.1c

1. Calculate the value of each of the following numbers in index form.

(a) 9^4

(b) $(-4)^5$

(c) $(2.5)^3$

(d) $(-3.2)^3$

(e) $\left(\dfrac{3}{8}\right)^5$

(f) $\left(-\dfrac{1}{6}\right)^4$

(g) $\left(1\dfrac{2}{3}\right)^2$

(h) $\left(-2\dfrac{1}{3}\right)^3$

1.2 Law of Indices

📝 **What is the relationship between multiplication of numbers in index form with the same base and repeated multiplication?**

LEARNING STANDARD
Relate the multiplication of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.

Brainstorming 1 🔭 *In pairs*

Aim: To identify the relationship between multiplication of numbers in index form with the same base and repeated multiplication.

Steps:
1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Exhibit three examples in the mathematics corner for other groups to give feedback.

Multiplication of numbers in index form	Repeated multiplication
(a) $2^3 \times 2^4$	$\overbrace{(2 \times 2 \times 2)}^{3\ \text{factors}} \times \overbrace{(2 \times 2 \times 2 \times 2)}^{4\ \text{factors}} = \overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}^{7\ \text{factors (overall)}} = 2^7$ $2^3 \times 2^4 = 2^7 \quad \boxed{7 = 3 + 4}$ $2^3 \times 2^4 = 2^{3+4}$
(b) $3^2 \times 3^3$	$\overbrace{(3 \times 3)}^{2\ \text{factors}} \times \overbrace{(3 \times 3 \times 3)}^{3\ \text{factors}} = \overbrace{3 \times 3 \times 3 \times 3 \times 3}^{5\ \text{factors (overall)}} = 3^5$ $3^2 \times 3^3 = 3^{\square}$ $3^2 \times 3^3 = 3^{\boxed{}}$

Multiplication of numbers in index form	Repeated multiplication
(c) $5^4 \times 5^2$	$\overbrace{(5 \times 5 \times 5 \times 5)}^{\text{4 factors}} \times \overbrace{(5 \times 5)}^{\text{2 factors}} = \overbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}^{\text{6 factors (overall)}} = 5^6$ $5^4 \times 5^2 = 5^{\square}$ $5^4 \times 5^2 = 5^{\boxed{}}$

Discussion:

What is your conclusion regarding the relationship between multiplication of numbers in index form and repeated multiplication?

From Brainstorming 1, it is found that;

$$2^3 \times 2^4 = 2^{3+4}$$
$$3^2 \times 3^3 = 3^{2+3}$$
$$5^4 \times 5^2 = 5^{4+2}$$

DISCUSSION CORNER

Given,
$$a^m \times a^n = b^m \times b^n.$$
Is $a = b$? Discuss.

In general, $\boxed{a^m \times a^n = a^{m+n}}$

Example 6

Simplify each of the following.

(a) $7^2 \times 7^3$ (b) $(0.2)^2 \times (0.2)^4 \times (0.2)^5$ (c) $2k^2 \times 4k^3$ (d) $3m^4 \times \dfrac{1}{6}m^5 \times 12m$

Solution:

(a) $7^2 \times 7^3$
$= 7^{2+3}$
$= 7^5$

(b) $(0.2)^2 \times (0.2)^4 \times (0.2)^5$
$= (0.2)^{2+4+5}$
$= (0.2)^{11}$

REMINDER

$$a = a^1$$

(c) $2k^2 \times 4k^3$
$= (2 \times 4)(k^2 \times k^3)$ → Operation of the coefficients
$= 8k^{2+3}$
$= 8k^5$

(d) $3m^4 \times \dfrac{1}{6}m^5 \times 12m$
$= (3 \times \dfrac{1}{6} \times 12)(m^4 \times m^5 \times m^1)$
$= 6m^{4+5+1}$
$= 6m^{10}$

SMART MIND

If $m^a \times m^b = m^8$, such that $a > 0$ and $b > 0$, what are the possible values of a and b?

MIND TEST 1.2a

1. Simplify each of the following.

(a) $3^2 \times 3 \times 3^4$

(b) $(-0.4)^4 \times (-0.4)^3 \times (-0.4)$

(c) $\left(\dfrac{4}{7}\right) \times \left(\dfrac{4}{7}\right)^3 \times \left(\dfrac{4}{7}\right)^5$

(d) $\left(-1\dfrac{2}{5}\right)^2 \times \left(-1\dfrac{2}{5}\right)^3 \times \left(-1\dfrac{2}{5}\right)^5$

(e) $4m^2 \times \dfrac{1}{2}m^3 \times (-3)m^4$

(f) $n^6 \times \dfrac{4}{25}n^2 \times \dfrac{5}{4}n^3 \times n$

(g) $-x^4 \times \dfrac{25}{4}x \times \dfrac{12}{5}x^2$

(h) $-\dfrac{1}{2}y^5 \times (-6)y^3 \times \dfrac{1}{3}y^4$

How do you simplify a number or an algebraic term in index form with different bases?

Example 7

Simplify each of the following.

(a) $m^3 \times n^2 \times m^4 \times n^5$

(b) $(0.3)^2 \times (0.2)^2 \times 0.3 \times (0.2)^5 \times (0.3)^3$

(c) $p^2 \times m^3 \times p^4 \times n^3 \times m^4 \times n^2$

(d) $-m^4 \times 2n^5 \times 3m \times \dfrac{1}{4}n^2$

Solution:

(a) $m^3 \times n^2 \times m^4 \times n^5$

$= m^3 \times m^4 \times n^2 \times n^5$ — Group the terms with the same base.

$= m^{3+4} \times n^{2+5}$ — Add the indices for terms with the same base.

$= m^7 \times n^7$

$= m^7 n^7$

(b) $(0.3)^2 \times (0.2)^2 \times 0.3 \times (0.2)^5 \times (0.3)^3$

$= (0.3)^2 \times (0.3)^1 \times (0.3)^3 \times (0.2)^2 \times (0.2)^5$

$= (0.3)^{(2+1+3)} \times (0.2)^{(2+5)}$

$= (0.3)^6 \times (0.2)^7$

(c) $p^2 \times m^3 \times p^4 \times n^3 \times m^4 \times n^2$

$= m^3 \times m^4 \times n^3 \times n^2 \times p^2 \times p^4$

$= m^{3+4} \times n^{3+2} \times p^{2+4}$

$= m^7 n^5 p^6$

(d) $-m^4 \times 2n^5 \times 3m \times \dfrac{1}{4}n^2$

$= (-1 \times 2 \times 3 \times \dfrac{1}{4}) m^4 \times m^1 \times n^5 \times n^2$

$= -\dfrac{3}{2}m^{4+1} n^{5+2}$

$= -\dfrac{3}{2}m^5 n^7$

MIND TEST 1.2b

1. State in simplest index form.

(a) $5^4 \times 9^3 \times 5 \times 9^2$

(b) $(0.4)^2 \times (1.2)^3 \times (0.4) \times (1.2)^5 \times (1.2)$

(c) $12x^5 \times y^3 \times \dfrac{1}{2} x \times \dfrac{2}{3} y^4$

(d) $-2k^5 \times p^6 \times \dfrac{1}{4} p^5 \times 3k$

What is the relationship between division of numbers in index form with the same base and repeated multiplication?

Brainstorming 2

In pairs

Aim: To identify the relationship between division of numbers in index form with the same base and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your findings.

Division of numbers in index form	Repeated multiplication
(a) $4^5 \div 4^2$	5 factors $\dfrac{4^5}{4^2} = \dfrac{4 \times 4 \times 4 \times 4 \times 4}{4 \times 4} = 4 \times 4 \times 4 = 4^3$ 2 factors 3 factors (Remainder) $4^5 \div 4^2 = 4^{\boxed{3}}$ $4^5 \div 4^2 = 4^{\boxed{5-2}}$ $\boxed{3 = 5-2}$
(b) $2^6 \div 2^2$	6 factors $\dfrac{2^6}{2^2} = \dfrac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2} = 2 \times 2 \times 2 \times 2 = 2^4$ 2 factors 4 factors (Remainder) $2^6 \div 2^2 = 2^{\boxed{}}$ $2^6 \div 2^2 = 2^{\boxed{}}$
(c) $(-3)^5 \div (-3)^3$	5 factors $\dfrac{(-3)^5}{(-3)^3} = \dfrac{(-3) \times (-3) \times (-3) \times (-3) \times (-3)}{(-3) \times (-3) \times (-3)} = (-3) \times (-3) = (-3)^2$ 3 factors 2 factors (Remainder) $(-3)^5 \div (-3)^3 = (-3)^{\boxed{}}$ $(-3)^5 \div (-3)^3 = (-3)^{\boxed{}}$

Discussion:

What is the relationship between division of numbers in index form and repeated multiplication?

From Brainstorming 2, it is found that;

$$4^5 \div 4^2 = 4^{5-2}$$
$$2^6 \div 2^2 = 2^{6-2}$$
$$(-3)^5 \div (-3)^3 = (-3)^{5-3}$$

In general, $\boxed{a^m \div a^n = a^{m-n}}$

SMART MIND

Given $m^{a-b} = m^7$ and $0 \leqslant a \leqslant 10$. If $a > b$, state the possible values of a and b.

Example 8

Simplify each of the following.

(a) $5^4 \div 5^2$ (b) $(-3)^4 \div (-3)^2 \div (-3)$ (c) $m^4 n^3 \div m^2 n$

(d) $25x^2 y^3 \div 5xy$ (e) $12m^{10} \div 4m^5 \div m^2$ (f) $-16p^8 \div 2p^5 \div 4p^2$

Solution:

(a) $5^4 \div 5^2$
$= 5^{4-2}$
$= 5^2$

(b) $(-3)^4 \div (-3)^2 \div (-3)$
$= (-3)^4 \div (-3)^2 \div (-3)^1$
$= (-3)^{4-2-1}$
$= (-3)^1$
$= -3$

(c) $m^4 n^3 \div m^2 n$
$= m^4 n^3 \div m^2 n^1$
$= m^{4-2} n^{3-1}$
$= m^2 n^2$

(d) $25x^2y^3 \div 5xy$

$= 25x^2y^3 \div 5x^1y^1$

$= \dfrac{25}{5}x^{2-1}\,y^{3-1}$ ← Operation of the coefficients

$= 5x^1y^2$

$= 5xy^2$

(e) $12m^{10} \div 4m^5 \div m^2$

$= \dfrac{12}{4}(m^{10} \div m^5 \div m^2)$

$= 3(m^{10-5}) \div m^2$

$= 3m^{5-2}$

$= 3m^3$

(f) $-16p^8 \div 2p^5 \div 4p^2$

$= \dfrac{-16}{2}(p^8 \div p^5) \div 4p^2$

$= -8p^{8-5} \div 4p^2$

$= -8p^3 \div 4p^2$

$= -\dfrac{8}{4}(p^3 \div p^2)$

$= -2p^{3-2}$

$= -2p^1$

$= -2p$

MIND TEST 1.2c

1. Simplify each of the following.

(a) $4^5 \div 4^4$

(b) $7^{10} \div 7^6 \div 7^2$

(c) $\dfrac{m^8n^6}{m^4n}$

(d) $\dfrac{27x^4y^5}{9x^3y^2}$

(e) $m^7 \div m^2 \div m^4$

(f) $-25h^4 \div 5h^2 \div h$

2. Copy and complete each of the following equations.

(a) $8^{\square} \div 8^4 \div 8^3 = 8$

(b) $m^4n^{\square} \div m^{\square}\,n^5 = m^2n$

(c) $\dfrac{m^{10}\,n^4 \times m^{\square}\,n^2}{m^7n} = m^5n^{\square}$

(d) $\dfrac{27x^3y^6 \times xy^{\square}}{\square x^2y^3} = 3x^{\square}\,y^5$

3. If $\dfrac{2^x \times 3^y}{2^4 \times 3^2} = 6$, determine the value of $x + y$.

✎ **What is the relationship between a number in index form raised to a power and repeated multiplication?**

Brainstorming 3 👀 👥 In pairs

Aim: To identify the relationship between a number in index form raised to a power and repeated multiplication.

Steps:

1. Study example (a) and complete examples (b) and (c).
2. Discuss with your friend and state three other examples.
3. Present your finding.

Index form raised to a power	Repeated multiplication in index form	Conclusion
(a) $(3^2)^4$	4 factors $3^2 \times 3^2 \times 3^2 \times 3^2$ $= 3^{2+2+2+2}$ (4 times) ← 2 is added 4 times $= 3^{2(4)}$	$(3^2)^4 = 3^{2(4)}$ $= 3^8$

Index form raised to a power	Repeated multiplication in index form	Conclusion
(b) $(5^4)^3$	3 factors $5^4 \times 5^4 \times 5^4$ $= 5^{4+4+4}$ ← 4 is added 3 times 3 times $= 5^{4(3)}$	$(5^4)^3 = 5^{\boxed{}}$ $= 5^{\boxed{}}$
(c) $(4^3)^6$	6 factors $4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$ $= 4^{3+3+3+3+3+3}$ ← 3 is added 6 times 6 times $= 4^{3(6)}$	$(4^3)^6 = 4^{\boxed{}}$ $= 4^{\boxed{}}$

Discussion:

What is your conclusion regarding the index form raised to a power and repeated multiplication in index form?

The conclusion in Brainstorming 3 can be checked using the following method.

Example (a)

$(3^2)^4 = 3^2 \times 3^2 \times 3^2 \times 3^2$
$\quad\quad = 3^{2+2+2+2}$
$\quad\quad = 3^8$
$3^{2(4)} = 3^{2 \times 4}$
$\quad\quad = 3^8$

Example (b)

$(5^4)^3 = 5^4 \times 5^4 \times 5^4$
$\quad\quad = 5^{4+4+4}$
$\quad\quad = 5^{12}$
$5^{4(3)} = 5^{4 \times 3}$
$\quad\quad = 5^{12}$

Example (c)

$(4^3)^6 = 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3 \times 4^3$
$\quad\quad = 4^{3+3+3+3+3+3}$
$\quad\quad = 4^{18}$
$4^{3(6)} = 4^{3 \times 6}$
$\quad\quad = 4^{18}$

From Brainstorming 3, it can be found that;

$(3^2)^4 = 3^{2(4)}$
$(5^4)^3 = 5^{4(3)}$
$(4^3)^6 = 4^{3(6)}$

In general, $\boxed{(a^m)^n = a^{mn}}$

SMART MIND

Given,
$$m^{rt} = 3^{12}$$
What are the possible values of m, r and t if $r > t$?

Example 9

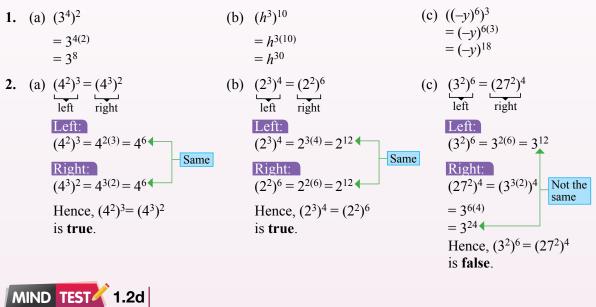1. Simplify each of the following.

   (a) $(3^4)^2$
   (b) $(h^3)^{10}$
   (c) $((-y)^6)^3$

2. Determine whether the following equations are **true** or **false**.

   (a) $(4^2)^3 = (4^3)^2$
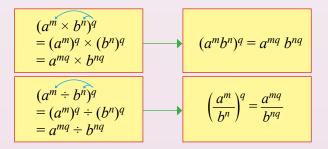   (b) $(2^3)^4 = (2^2)^6$
   (c) $(3^2)^6 = (27^2)^4$

11

KPM

Solution:

1. (a) $(3^4)^2$

$= 3^{4(2)}$

$= 3^8$

(b) $(h^3)^{10}$

$= h^{3(10)}$

$= h^{30}$

(c) $((-y)^6)^3$

$= (-y)^{6(3)}$

$= (-y)^{18}$

2. (a) $(4^2)^3 = (4^3)^2$

  left   right

Left:
$(4^2)^3 = 4^{2(3)} = 4^6$

Same

Right:
$(4^3)^2 = 4^{3(2)} = 4^6$

Hence, $(4^2)^3 = (4^3)^2$
is **true**.

(b) $(2^3)^4 = (2^2)^6$

  left   right

Left:
$(2^3)^4 = 2^{3(4)} = 2^{12}$

Same

Right:
$(2^2)^6 = 2^{2(6)} = 2^{12}$

Hence, $(2^3)^4 = (2^2)^6$
is **true**.

(c) $(3^2)^6 = (27^2)^4$

  left   right

Left:
$(3^2)^6 = 3^{2(6)} = 3^{12}$

Right:
$(27^2)^4 = (3^{3(2)})^4$ Not the same
$= 3^{6(4)}$
$= 3^{24}$

Hence, $(3^2)^6 = (27^2)^4$
is **false**.

MIND TEST **1.2d**

1. Use law of indices to simplify each of the following statements.

(a) $(12^5)^2$

(b) $(3^{10})^2$

(c) $(7^2)^3$

(d) $((-4)^3)^7$

(e) $(k^8)^3$

(f) $(g^2)^{13}$

(g) $((-m)^4)^3$

(h) $((-c)^7)^3$

2. Determine whether the following equations are **true** or **false**.

(a) $(2^4)^5 = (2^2)^{10}$

(b) $(3^3)^7 = (27^2)^4$

(c) $(5^2)^5 = (125^2)^3$

(d) $-(7^2)^4 = (-49^2)^3$

How do you use law of indices to perform operations of multiplication and division?

$(a^m \times b^n)^q$
$= (a^m)^q \times (b^n)^q$
$= a^{mq} \times b^{nq}$

→ $(a^m b^n)^q = a^{mq} b^{nq}$

$(a^m \div b^n)^q$
$= (a^m)^q \div (b^n)^q$
$= a^{mq} \div b^{nq}$

→ $\left(\dfrac{a^m}{b^n}\right)^q = \dfrac{a^{mq}}{b^{nq}}$

Example 10

1. Simplify each of the following.

(a) $(7^3 \times 5^4)^3$

(b) $(2^4 \times 5^3 \times 11^2)^5$

(c) $(p^2 q^3 r)^4$

(d) $(5m^4 n^3)^2$

(e) $\left(\dfrac{2^5}{3^2}\right)^4$

(f) $\left(\dfrac{2x^3}{3y^7}\right)^4$

(g) $\dfrac{(3m^2 n^3)^3}{6m^3 n}$

(h) $\dfrac{(2x^3 y^4)^4 \times (3xy^2)^3}{36x^{10} y^{12}}$

CHAPTER 1

Solution:

(a) $(7^3 \times 5^4)^3$
$= 7^{3(3)} \times 5^{4(3)}$
$= 7^9 \times 5^{12}$

(b) $(2^4 \times 5^3 \times 11^2)^5$
$= 2^{4(5)} \times 5^{3(5)} \times 11^{2(5)}$
$= 2^{20} \times 5^{15} \times 11^{10}$

(c) $(p^2 q^3 r)^4$
$= p^{2(4)} q^{3(4)} r^{1(4)}$
$= p^8 q^{12} r^4$

(d) $(5m^4 n^3)^2$
$= 5^2 m^{4(2)} n^{3(2)}$
$= 25 m^8 n^6$

(e) $\left(\dfrac{2^5}{3^2}\right)^4$
$= \dfrac{2^{5(4)}}{3^{2(4)}}$
$= \dfrac{2^{20}}{3^8}$

(f) $\left(\dfrac{2x^3}{3y^7}\right)^4$
$= \dfrac{2^4 x^{3(4)}}{3^4 y^{7(4)}}$
$= \dfrac{16x^{12}}{81y^{28}}$

(g) $\dfrac{(3m^2 n^3)^3}{6m^3 n}$
$= \dfrac{3^3 m^{2(3)} n^{3(3)}}{6m^3 n^1}$
$= \dfrac{27 m^6 n^9}{6m^3 n^1}$
$= \dfrac{9}{2} m^{6-3} n^{9-1}$
$= \dfrac{9}{2} m^3 n^8$

(h) $\dfrac{(2x^3 y^4)^4 \times (3xy^2)^3}{36 x^{10} y^{12}}$
$= \dfrac{2^4 x^{3(4)} y^{4(4)} \times 3^3 x^{1(3)} y^{2(3)}}{36 x^{10} y^{12}}$
$= \dfrac{16 x^{12} y^{16} \times 27 x^3 y^6}{36 x^{10} y^{12}}$
$= \left(\dfrac{16 \times 27}{36}\right) x^{12+3-10} y^{16+6-12}$
$= 12 x^5 y^{10}$

FLASHBACK

$a^m \times a^n = a^{m+n}$
$a^m \div a^n = a^{m-n}$
$(a^m)^n = a^{mn}$

QUIZ ✓

$m^m = 256$.
What is the value of m?

DISCUSSION CORNER 💬

Why is $1^n = 1$ for all values of n?

Discuss.

MIND TEST ✏ **1.2e**

1. Simplify each of the following.
(a) $(2 \times 3^4)^2$
(b) $(11^3 \times 9^5)^3$
(c) $(13^3 \div 7^6)^2$
(d) $(5^3 \times 3^4)^5$
(e) $(m^3 n^4 p^2)^5$
(f) $(2w^2 x^3)^4$
(g) $\left(\dfrac{-3a^5}{b^4}\right)^6$
(h) $\left(\dfrac{2a^5}{3b^4}\right)^3$

2. Simplify each of the following.
(a) $\left(\dfrac{11^3 \times 4^2}{11^2}\right)^2$
(b) $\dfrac{3^3 \times (6^2)^3}{6^4}$
(c) $\left(\dfrac{4^2}{6^3}\right)^3 \div \dfrac{4^2}{6^3}$
(d) $\dfrac{((-4)^6)^2 \times (-5^2)^3}{(-4)^6 \times (-5)^2}$
(e) $\dfrac{x^2 y^6 \times x^3}{xy^2}$
(f) $\dfrac{(h^3 k^2)^4}{(hk)^2}$
(g) $\dfrac{(m^5 n^7)^3}{(m^2 n^3)^2}$
(h) $\dfrac{(b^2 d^4)^3}{(b^2 d^3)^2}$

3. Simplify each of the following.
(a) $\dfrac{(2m^2 n^4)^3 \times (3mn^4)^2}{12m^7 n^{12}}$
(b) $\dfrac{(5xy^4)^2 \times 6x^{10}y}{15x^4 y^6}$
(c) $\dfrac{24d^3 e^5 \times (3d^3 e^4)^2}{(d^5 e^6) \times (6de^2)^3}$

✏️ **How do you verify $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n}$; $a \neq 0$?**

Brainstorming 4 👀
In pairs

Aim: To determine the value of a number or an algebraic term with a zero index.

Steps:
1. Study and complete the following table.
2. What is your conclusion regarding zero index?

Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^3$	$2^{3-3} = 2^0$	$\dfrac{2 \times 2 \times 2}{2 \times 2 \times 2} = 1$	$2^0 = 1$
(d) $m^5 \div m^5$	$m^{5-5} = m^0$	$\dfrac{m \times m \times m \times m \times m}{m \times m \times m \times m \times m} = 1$	$m^0 = 1$
(c) $5^4 \div 5^4$			
(d) $(-7)^2 \div (-7)^2$			
(e) $n^6 \div n^6$			

Discussion:
1. Are your answers similar with other groups?
2. What is your conclusion regarding zero index?

From Brainstorming 4, it is found that;

$$2^0 = 1$$
$$m^0 = 1$$

Therefore, a number or an algebraic term with a zero index will give a value of 1.

In general, $\boxed{a^0 = 1 \; ; \; a \neq 0}$

🧮 **How do you verify $a^{-n} = \dfrac{1}{a^n}$?**

Brainstorming 5 👀
In groups

Aim: To verify $a^{-n} = \dfrac{1}{a^n}$.

Steps:
1. Study and complete the following table.
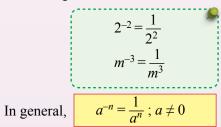
Division in index form	Solution		Conclusion from the solution
	Law of indices	Repeated multiplication	
(a) $2^3 \div 2^5$	$2^{3-5} = 2^{-2}$	$\dfrac{\cancel{2} \times \cancel{2} \times \cancel{2}}{2 \times 2 \times \cancel{2} \times \cancel{2} \times \cancel{2}} = \dfrac{1}{2 \times 2} = \dfrac{1}{2^2}$	$2^{\boxed{-2}} = \dfrac{1}{2^{\boxed{2}}}$
(b) $m^2 \div m^5$	$m^{2-5} = m^{-3}$	$\dfrac{\cancel{m} \times \cancel{m}}{m \times m \times m \times \cancel{m} \times \cancel{m}} = \dfrac{1}{m \times m \times m} = \dfrac{1}{m^3}$	$m^{\boxed{-3}} = \dfrac{1}{m^{\boxed{3}}}$
(c) $3^2 \div 3^6$			
(d) $(-4)^3 \div (-4)^7$			
(e) $p^4 \div p^8$			

Discussion:
1. Are your answers similar with other groups?
2. What is your conclusion?

From Brainstorming 5, it is found that;

$$2^{-2} = \dfrac{1}{2^2}$$
$$m^{-3} = \dfrac{1}{m^3}$$

In general, $\boxed{a^{-n} = \dfrac{1}{a^n} \; ; \; a \neq 0}$

Example 11

1. State each of the following terms in positive index form.

(a) a^{-2} (b) x^{-4} (c) $\dfrac{1}{8^{-5}}$

(d) $\dfrac{1}{y^{-9}}$ (e) $2m^{-3}$ (f) $\dfrac{3}{5} n^{-8}$

(g) $\left(\dfrac{2}{3}\right)^{-10}$ (h) $\left(\dfrac{x}{y}\right)^{-7}$

2. State each of the following in negative index form.

(a) $\dfrac{1}{3^4}$ (b) $\dfrac{1}{m^5}$ (c) 7^5

(d) n^{20} (e) $\left(\dfrac{4}{5}\right)^8$ (f) $\left(\dfrac{m}{n}\right)^{15}$

3. Simplify each of the following.
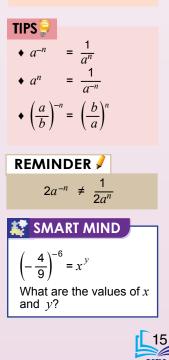
(a) $3^2 \times 3^4 \div 3^8$ (b) $\dfrac{(2^4)^2 \times (3^5)^3}{(2^8 \times 3^6)^2}$ (c) $\dfrac{(4xy^2)^2 \times x^5y}{(2x^3y)^5}$

Scan the QR Code or visit http://bukutekskssm.my/Mathematics/F3/Chapter1 AlternativeMethod.mp4 to watch a video that describes alternative method to verify $a^{-1} = \dfrac{1}{a^n}$.

BULLETIN

Negative index is a number or an algebraic term that has an index of a negative value.

TIPS

♦ $a^{-n} = \dfrac{1}{a^n}$

♦ $a^n = \dfrac{1}{a^{-n}}$

♦ $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$

REMINDER

$2a^{-n} \neq \dfrac{1}{2a^n}$

SMART MIND

$\left(-\dfrac{4}{9}\right)^{-6} = x^y$

What are the values of x and y?

Solution:

1. (a) $a^{-2} = \dfrac{1}{a^2}$ (b) $x^{-4} = \dfrac{1}{x^4}$ (c) $\dfrac{1}{8^{-5}} = 8^5$ (d) $\dfrac{1}{y^{-9}} = y^9$

(e) $2m^{-3} = \dfrac{2}{m^3}$ (f) $\dfrac{3}{5}n^{-8} = \dfrac{3}{5n^8}$ (g) $\left(\dfrac{2}{3}\right)^{-10} = \left(\dfrac{3}{2}\right)^{10}$ (h) $\left(\dfrac{x}{y}\right)^{-7} = \left(\dfrac{y}{x}\right)^{7}$

2. (a) $\dfrac{1}{3^4} = 3^{-4}$ (b) $\dfrac{1}{m^5} = m^{-5}$ (c) $7^5 = \dfrac{1}{7^{-5}}$ (d) $n^{20} = \dfrac{1}{n^{-20}}$

(e) $\left(\dfrac{4}{5}\right)^{8} = \left(\dfrac{5}{4}\right)^{-8}$ (f) $\left(\dfrac{m}{n}\right)^{15} = \left(\dfrac{n}{m}\right)^{-15}$

3. (a) $3^2 \times 3^4 \div 3^8$
$= 3^{2+4-8}$
$= 3^{-2}$
$= \dfrac{1}{3^2}$

(b) $\dfrac{(2^4)^2 \times (3^5)^3}{(2^8 \times 3^6)^2}$
$= \dfrac{2^8 \times 3^{15}}{2^{16} \times 3^{12}}$
$= 2^{8-16} \times 3^{15-12}$
$= 2^{-8} \times 3^3$
$= \dfrac{3^3}{2^8}$

(c) $\dfrac{(4xy^2)^2 \times x^5y}{(2x^3y)^5}$
$= \dfrac{4^2x^2y^4 \times x^5y^1}{2^5x^{15}y^5}$
$= \dfrac{16}{32}x^{2+5-15}\,y^{4+1-5}$
$= \dfrac{1}{2}x^{-8}y^0$
$= \dfrac{1}{2x^8}$

TIPS
$y^0 = 1$
$y^1 = y$

MIND TEST 1.2f

1. State each of the following terms in positive index form.

(a) 5^{-3} (b) 8^{-4} (c) x^{-8} (d) y^{-16} (e) $\dfrac{1}{a^{-4}}$

(f) $\dfrac{1}{20^{-2}}$ (g) $3n^{-4}$ (h) $-5n^{-6}$ (i) $\dfrac{2}{7}m^{-5}$ (j) $\left(-\dfrac{3}{8}\right)m^{-4}$

(k) $\left(\dfrac{2}{5}\right)^{-12}$ (l) $\left(-\dfrac{3}{7}\right)^{-14}$ (m) $\left(\dfrac{x}{y}\right)^{-10}$ (n) $\left(\dfrac{2x}{3y}\right)^{-4}$ (o) $\left(\dfrac{1}{2x}\right)^{-5}$

2. State each of the following terms in negative index form.

(a) $\dfrac{1}{5^4}$ (b) $\dfrac{1}{8^3}$ (c) $\dfrac{1}{m^7}$ (d) $\dfrac{1}{n^9}$ (e) 10^2

(f) $(-4)^3$ (g) m^{12} (h) n^{16} (i) $\left(\dfrac{4}{7}\right)^{9}$ (j) $\left(\dfrac{x}{y}\right)^{10}$

3. Simplify each of the following.

(a) $\dfrac{(4^2)^3 \times 4^5}{(4^6)^2}$ (b) $\dfrac{(2^3 \times 3^2)^3}{(2 \times 3^4)^5}$ (c) $\dfrac{(5^2)^5}{(2^3)^{-2} \times (5^4)^2}$

(d) $\dfrac{3m^2n^4 \times (mn^3)^{-2}}{9m^3n^5}$ (e) $\dfrac{(2m^2n^2)^{-3} \times (3mn^2)^4}{(9m^3n)^2}$ (f) $\dfrac{(4m^2n^4)^2}{(2m^{-2}n)^5 \times (3m^4n)^2}$

How do you determine and state the relationship between fractional indices and roots and powers?

Relationship between $\sqrt[n]{a}$ and $a^{\frac{1}{n}}$

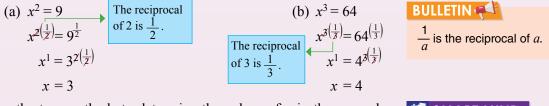In Form 1, you have learnt about square and square root as well as cube and cube root. Determine the value of x for

    (a) $x^2 = 9$ (b) $x^3 = 64$

TIPS

♦ $9 = 3^2$ ♦ $64 = 4^3$

Solution:

(a) $x^2 = 9$ Square roots are used to eliminate squares.

$\sqrt{x^2} = \sqrt{3^2}$

$x = 3$

(b) $x^3 = 64$

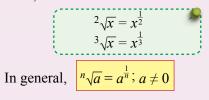$\sqrt[3]{x^3} = \sqrt[3]{4^3}$ Cube roots are used to eliminate cubes.

$x = 4$

Did you know that the values of x in examples (a) and (b) above can be determined by raising the index to the power of its reciprocal?

(a) $x^2 = 9$ The reciprocal of 2 is $\frac{1}{2}$.

$x^{2\left(\frac{1}{2}\right)} = 9^{\frac{1}{2}}$

$x^1 = 3^{2\left(\frac{1}{2}\right)}$

$x = 3$

(b) $x^3 = 64$

$x^{3\left(\frac{1}{3}\right)} = 64^{\left(\frac{1}{3}\right)}$

$x^1 = 4^{3\left(\frac{1}{3}\right)}$ The reciprocal of 3 is $\frac{1}{3}$.

$x = 4$

BULLETIN

$\frac{1}{a}$ is the reciprocal of a.

From the two methods to determine the values of x in the examples above, it is found that;

$$2\sqrt{x} = x^{\frac{1}{2}}$$
$$3\sqrt{x} = x^{\frac{1}{3}}$$

SMART MIND

What is the solution for $\sqrt{-4}$? Discuss.

In general, $\sqrt[n]{a} = a^{\frac{1}{n}}\,;\ a \neq 0$

Example 12

1. Convert each of the following terms into the form $a^{\frac{1}{n}}$.

  (a) $\sqrt[2]{36}$ (b) $\sqrt[3]{-27}$ (c) $\sqrt[5]{m}$ (d) $\sqrt[7]{n}$

2. Convert each of the following terms into the form $\sqrt[n]{a}$.

  (a) $125^{\frac{1}{5}}$ (b) $256^{\frac{1}{8}}$ (c) $(-1\,000)^{\frac{1}{3}}$ (d) $n^{\frac{1}{12}}$

3. Calculate the value of each of the following terms.

  (a) $\sqrt[5]{-32}$ (b) $\sqrt[6]{729}$ (c) $512^{\frac{1}{3}}$ (d) $(-243)^{\frac{1}{5}}$

Solution:

1. (a) $\sqrt[2]{36} = 36^{\frac{1}{2}}$ (b) $\sqrt[3]{-27} = (-27)^{\frac{1}{3}}$ (c) $\sqrt[5]{m} = m^{\frac{1}{5}}$ (d) $\sqrt[7]{n} = n^{\frac{1}{7}}$

2. (a) $125^{\frac{1}{5}} = \sqrt[5]{125}$ (b) $256^{\frac{1}{8}} = \sqrt[8]{256}$ (c) $(-1\,000)^{\frac{1}{3}} = \sqrt[3]{(-1\,000)}$ (d) $n^{\frac{1}{12}} = \sqrt[12]{n}$

3. (a) $^5\sqrt{-32} = (-32)^{\frac{1}{5}}$ (b) $^6\sqrt{729} = 729^{\frac{1}{6}}$ (c) $512^{\frac{1}{3}} = 8^{3\left(\frac{1}{3}\right)}$ (d) $(-243)^{\frac{1}{5}} = (-3)^{5\left(\frac{1}{5}\right)}$

$\quad\quad = (-2)^{5\left(\frac{1}{5}\right)}$ $\quad\quad = 3^{6\left(\frac{1}{6}\right)}$ $\quad\quad = 8^1$ $\quad\quad = (-3)^1$

$\quad\quad = (-2)^1$ $\quad\quad = 3^1$ $\quad\quad = 8$ $\quad\quad = -3$

$\quad\quad = -2$ $\quad\quad = 3$

TIPS

You can use a scientific calculator to check the answers.

MIND TEST 1.2g

1. Convert each of the following terms into the form $a^{\frac{1}{n}}$.

(a) $^3\sqrt{125}$ (b) $^7\sqrt{2\ 187}$ (c) $^5\sqrt{-1\ 024}$ (d) $^{10}\sqrt{n}$

2. Convert each of the following terms into the form $^n\sqrt{a}$.

(a) $4^{\frac{1}{2}}$ (b) $32^{\frac{1}{5}}$ (c) $(-729)^{\frac{1}{3}}$ (d) $n^{\frac{1}{15}}$

3. Calculate the value of each of the following terms.

(a) $^3\sqrt{343}$ (b) $^5\sqrt{-7\ 776}$ (c) $262\ 144^{\frac{1}{6}}$ (d) $(-32\ 768)^{\frac{1}{5}}$

What is the relationship between $a^{\frac{m}{n}}$ and $(a^m)^{\frac{1}{n}}$, $(a^{\frac{1}{n}})^m$, $^n\sqrt{a^m}$ dan $(^n\sqrt{a})^m$?

You have learnt that;

$$a^{mn} = (a^m)^n \text{ and } ^n\sqrt{a^1} = a^{\frac{1}{n}}$$

From the two laws of indices above, we can convert $a^{\frac{m}{n}}$ into $(a^m)^{\frac{1}{n}}$, $(a^{\frac{1}{n}})^m$, $^n\sqrt{a^m}$ and $(^n\sqrt{a})^m$.
Calculate the value of each of the following. Complete the table as shown in example (a).

	$a^{\frac{m}{n}}$	$(a^m)^{\frac{1}{n}}$	$(a^{\frac{1}{n}})^m$	$^n\sqrt{a^m}$	$(^n\sqrt{a})^m$
(a)	$64^{\frac{2}{3}}$	$(64^2)^{\frac{1}{3}}$ $= 4\ 096^{\left(\frac{1}{3}\right)}$ $= 16^{3\left(\frac{1}{3}\right)}$ $= 16$	$(64^{\frac{1}{3}})^2$ $= 4^{3\left(\frac{1}{3}\right)(2)}$ $= 4^2$ $= 16$	$^3\sqrt{64^2}$ $= ^3\sqrt{4\ 096}$ $= 16$	$(^3\sqrt{64})^2$ $= 4^2$ $= 16$
(b)	$16^{\frac{3}{4}}$				
(c)	$243^{\frac{2}{5}}$				

Are your answers in (b) and (c) the same when you use different index forms? Discuss.

From the activity above, it is found that;

$$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$$
$$a^{\frac{m}{n}} = ^n\sqrt{a^m} = (^n\sqrt{a})^m$$

CHAPTER 1

Example 13

1. Convert each of the following into the form $(a^m)^{\frac{1}{n}}$ and $(a^{\frac{1}{n}})^m$.

   (a) $81^{\frac{3}{2}}$ (b) $27^{\frac{2}{3}}$ (c) $h^{\frac{3}{5}}$

2. Convert each of the following into the form $\sqrt[n]{a^m}$ and $(\sqrt[n]{a})^m$.

   (a) $343^{\frac{2}{3}}$ (b) $4\,096^{\frac{5}{6}}$ (c) $m^{\frac{2}{5}}$

Solution:

1. (a) $81^{\frac{3}{2}} = (81^3)^{\frac{1}{2}}$ (b) $27^{\frac{2}{3}} = (27^2)^{\frac{1}{3}}$ (c) $h^{\frac{3}{5}} = (h^3)^{\frac{1}{5}}$

       $81^{\frac{3}{2}} = (81^{\frac{1}{2}})^3$ $27^{\frac{2}{3}} = (27^{\frac{1}{3}})^2$ $h^{\frac{3}{5}} = (h^{\frac{1}{5}})^3$

2. (a) $343^{\frac{2}{3}} = \sqrt[3]{343^2}$ (b) $4\,096^{\frac{5}{6}} = \sqrt[6]{4\,096^5}$ (c) $m^{\frac{2}{5}} = \sqrt[5]{m^2}$

       $343^{\frac{2}{3}} = (\sqrt[3]{343})^2$ $4\,096^{\frac{5}{6}} = (\sqrt[6]{4\,096})^5$ $m^{\frac{2}{5}} = (\sqrt[5]{m})^2$

MIND TEST 1.2h

1. Complete the following table.

$a^{\frac{m}{n}}$	$729^{\frac{5}{6}}$	$121^{\frac{3}{2}}$	$w^{\frac{3}{7}}$	$x^{\frac{2}{5}}$	$\left(\frac{16}{81}\right)^{\frac{3}{4}}$	$\left(\frac{h}{k}\right)^{\frac{2}{3}}$
$(a^m)^{\frac{1}{n}}$						
$(a^{\frac{1}{n}})^m$						
$\sqrt[n]{a^m}$						
$(\sqrt[n]{a})^m$						

Example 14

1. Calculate the value of each of the following.

   (a) $9^{\frac{5}{2}}$ (b) $16^{\frac{5}{4}}$

Solution:

1. (a) $9^{\frac{5}{2}}$ (b) $16^{\frac{5}{4}}$

   Method 1 $9^{\frac{5}{2}} = (\sqrt{9})^5 = (3)^5 = 243$ **Method 1** $16^{\frac{5}{4}} = (\sqrt[4]{16})^5 = 2^5 = 32$

   Method 2 $9^{\frac{5}{2}} = \sqrt{9^5} = \sqrt{59\,049} = 243$ **Method 2** $16^{\frac{5}{4}} = \sqrt[4]{16^5} = \sqrt[4]{1\,048\,576} = 32$

MIND TEST 1.2i

1. Calculate the value of each of the following..

(a) $27^{\frac{2}{3}}$ (b) $32^{\frac{2}{5}}$ (c) $128^{\frac{2}{7}}$ (d) $256^{\frac{3}{8}}$

(e) $64^{\frac{4}{3}}$ (f) $1\,024^{\frac{2}{5}}$ (g) $1\,296^{\frac{3}{4}}$ (h) $49^{\frac{3}{2}}$

(i) $2\,401^{\frac{1}{4}}$ (j) $121^{\frac{3}{2}}$ (k) $2\,197^{\frac{2}{3}}$ (l) $10\,000^{\frac{3}{4}}$

2. Complete the following diagrams with correct values.

(a)

$\square \sqrt{6\,561}\square$

$27^{\frac{\square}{3}}$ $3^{\square}$

81

$243^{\frac{4}{\square}}$ $9^{\square}$

$81^{\square}$

(b)

$25^{\frac{3}{\square}}$

$5^{\square}$ $125^{\square}$

125

$3\,125^{\frac{3}{\square}}$ $625^{\frac{\square}{4}}$

$\square \sqrt{15\,625}\square$

✎ How do you perform operations involving laws of indices?

Law of indices		
$a^m \times a^n = a^{m+n}$ $a^m \div a^n = a^{m-n}$ $(a^m)^n = a^{mn}$	$a^0 = 1$ $a^{-n} = \dfrac{1}{a^n}$	$a^{\frac{1}{n}} = \sqrt[n]{a}$ $a^{\frac{m}{n}} = a^{m\left(\frac{1}{n}\right)} = \left(a^{\frac{1}{n}}\right)^m$ $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Example 15

1. Simplify each of the following.

(a) $\dfrac{(-3x)^3 \times (2x^3 y^{-4})^2}{108x^4 y^3}$

(b) $\dfrac{\sqrt{m}\, n^{\frac{3}{4}} \times (mn^3)^{\frac{1}{3}}}{(m^{-1} \sqrt{n^3})^{\frac{1}{6}}}$

(c) $\dfrac{(2h)^2 \times (16h^8)^{\frac{1}{4}}}{(8^{\frac{1}{3}}h)^{-2}}$

Solution:

(a) $\dfrac{(-3x)^3 \times (2x^3 y^{-4})^2}{108x^4 y^3}$

$= \dfrac{(-3)^3 x^3 \times 2^2 x^{3(2)} y^{-4(2)}}{108x^4 y^3}$

$= \dfrac{-27x^3 \times 4x^6 y^{-8}}{108x^4 y^3}$

$= \left(\dfrac{-27 \times 4}{108}\right) x^{3+6-4} y^{-8-3}$

$= -1\, x^5 y^{-11}$

$= -\dfrac{x^5}{y^{11}}$

(b) $\dfrac{\sqrt{m}\, n^{\frac{3}{4}} \times (mn^3)^{\frac{1}{3}}}{(m^{-1} \sqrt{n^3})^{\frac{1}{6}}}$

$= \dfrac{m^{\frac{1}{2}} n^{\frac{3}{4}} \times m^{\frac{1}{3}} n^{3\left(\frac{1}{3}\right)}}{m^{-1\left(\frac{1}{6}\right)} n^{\frac{3}{2}\left(\frac{1}{6}\right)}}$

$= \dfrac{m^{\frac{1}{2}} n^{\frac{3}{4}} \times m^{\frac{1}{3}} n^1}{m^{-\frac{1}{6}} n^{\frac{1}{4}}}$

$= m^{\frac{1}{2} + \frac{1}{3} - \left(-\frac{1}{6}\right)} n^{\frac{3}{4} + 1 - \frac{1}{4}}$

$= m^1 n^{\frac{3}{2}}$

$= mn^{\frac{3}{2}}$

(c) $\dfrac{(2h)^2 \times (16h^8)^{\frac{1}{4}}}{(8^{\frac{1}{3}}h)^{-2}}$

$= \dfrac{2^2 h^2 \times 16^{\frac{1}{4}} h^{8\left(\frac{1}{4}\right)}}{8^{\frac{1}{3}(-2)} h^{(-2)}}$

$= \dfrac{2^2 h^2 \times 2^{4\left(\frac{1}{4}\right)} h^{8\left(\frac{1}{4}\right)}}{2^{3\left(\frac{1}{3}\right)(-2)} h^{(-2)}}$

$= \dfrac{2^2 h^2 \times 2^1 h^2}{2^{-2} h^{-2}}$

$= 2^{2+1-(-2)} h^{2+2-(-2)}$

$= 2^5 h^6$

$= 32\, h^6$

Example 16

1. Calculate the value of each of the following.

(a) $\dfrac{49^{\frac{1}{2}} \times 125^{-\frac{1}{3}}}{\sqrt[4]{2\,401} \times \sqrt[5]{3\,125}}$

(b) $\dfrac{16^{\frac{3}{4}} \times 81^{-\frac{1}{4}}}{(2^6 \times 3^4)^{\frac{1}{2}}}$

(c) $\dfrac{(243^{\frac{4}{5}} \times 5^{\frac{3}{2}})^2}{\sqrt[4]{81} \times \sqrt{25^4}}$

Solution:

(a) $\dfrac{49^{\frac{1}{2}} \times 125^{-\frac{1}{3}}}{\sqrt[4]{2\,401} \times \sqrt[5]{3\,125}}$

$= \dfrac{7^{2(\frac{1}{2})} \times 5^{3(-\frac{1}{3})}}{(7^4)^{\frac{1}{4}} \times (5^5)^{\frac{1}{5}}}$

$= \dfrac{7^1 \times 5^{-1}}{7^1 \times 5^1}$

$= 7^{1-1} \times 5^{-1-1}$

$= 7^0 \times 5^{-2}$

$= 1 \times \dfrac{1}{5^2}$

$= \dfrac{1}{25}$

(b) $\dfrac{16^{\frac{3}{4}} \times 81^{-\frac{1}{4}}}{(2^6 \times 3^4)^{\frac{1}{2}}}$

$= \dfrac{2^{4(\frac{3}{4})} \times 3^{4(-\frac{1}{4})}}{2^{6(\frac{1}{2})} \times 3^{4(\frac{1}{2})}}$

$= \dfrac{2^3 \times 3^{-1}}{2^3 \times 3^2}$

$= 2^{3-3} \times 3^{-1-2}$

$= 2^0 \times 3^{-3}$

$= 1 \times \dfrac{1}{3^3}$

$= \dfrac{1}{27}$

(c) $\dfrac{(243^{\frac{4}{5}} \times 5^{\frac{3}{2}})^2}{\sqrt[4]{81} \times \sqrt{25^4}}$

$= \dfrac{243^{\frac{4}{5}(2)} \times 5^{\frac{3}{2}(2)}}{81^{\frac{1}{4}} \times 25^{\frac{4}{2}}}$

$= \dfrac{3^{5(\frac{8}{5})} \times 5^3}{3^{4(\frac{1}{4})} \times 5^{2(\frac{4}{2})}}$

$= \dfrac{3^8 \times 5^3}{3^1 \times 5^4}$

$= 3^{8-1} \times 5^{3-4}$

$= 3^7 \times 5^{-1}$

$= \dfrac{3^7}{5}$

$= \dfrac{2\,187}{5}$

$= 437\dfrac{2}{5}$

MIND TEST 1.2j

1. Simplify each of the following.

(a) $\dfrac{\sqrt[3]{c^2 d^3 e} \times c^{\frac{1}{3}} d^2 e^{\frac{2}{3}}}{(c^{-3} d^2 e)^2}$

(b) $\dfrac{(mn^2)^3 \times (\sqrt{mn})^4}{(m^6 n^3)^{\frac{2}{3}}}$

(c) $\dfrac{\sqrt{25x^3 yz^2} \times 4x^2 z}{\sqrt{36x^5 yz^8}}$

2. Calculate the value of each of the following..

(a) $\dfrac{\sqrt{7^{-4} \times 11^4}}{49 \times 121}$

(b) $\dfrac{(5^{-3} \times 3^6)^{\frac{1}{3}} \times \sqrt[4]{16}}{(125 \times 729 \times 64)^{-\frac{1}{3}}}$

(c) $\dfrac{(2^6 \times 3^4 \times 5^2)^{\frac{3}{2}}}{\sqrt[4]{256} \times \sqrt{729} \times \sqrt[3]{125}}$

(d) $\dfrac{\sqrt[9]{512} \times \sqrt[3]{343} \times \sqrt{121}}{(64)^{\frac{1}{3}} \times (81)^{\frac{3}{4}} \times (14\,641)^{\frac{1}{4}}}$

(e) $\dfrac{(2^4 \times 3^6)^{\frac{1}{2}} \times \sqrt[3]{8} \times \sqrt{81}}{16^{\frac{3}{4}} \times 27^{\frac{1}{3}}}$

(f) $\dfrac{64^{\frac{2}{3}} \times \sqrt[3]{125} \times (2 \times \frac{1}{5})^{-3}}{4^2 \times \sqrt[4]{625}}$

3. Given $m = 2$ and $n = -3$, calculate the value of $64^{\frac{m}{3}} \times 512^{(-\frac{1}{n})} \div 81^{\frac{n}{2m}}$.

4. Given $a = \dfrac{1}{2}$ and $b = \dfrac{2}{3}$, calculate the value of $144^a \div 64^b \times 256^{-\frac{a}{b}}$.

✎ **How do you solve problems involving laws of indices?**

FLASHBACK

Common prime factors of 6 and 12 are 2 and 3.

Example 17

Calculate the value of $\sqrt{3} \times 12^{\frac{3}{2}} \div 6$ without using a calculator.

Understanding the problem

Calculate the value of numbers given in index form with different bases.

Planning a strategy

Convert each base into prime factors and calculate the value by applying laws of indices.

Implementing the strategy

$\sqrt{3} \times 12^{\frac{3}{2}} \div 6$

$= 3^{\frac{1}{2}} \times (2 \times 2 \times 3)^{\frac{3}{2}} \div (2 \times 3)$

$= 3^{\frac{1}{2}} \times 2^{\frac{3}{2}} \times 2^{\frac{3}{2}} \times 3^{\frac{3}{2}} \div (2^1 \times 3^1)$

$= 3^{\frac{1}{2} + \frac{3}{2} - 1} \times 2^{\frac{3}{2} + \frac{3}{2} - 1}$

$= 3^1 \times 2^2$

$= 12$

Making a conclusion

$\sqrt{3} \times 12^{\frac{3}{2}} \div 6 = 12$

Example 18

Calculate the value of x for the equation $3^x \times 9^{x+5} \div 3^4 = 1$.

Understanding the problem

Calculate the value of variable x which is part of the indices indeks.

Planning a strategy

The question is an equation. Hence, the value on the left side of the equation is the same as the value on the right side of the equation. Convert all the terms into index form with base of 3.

REMINDER

♦ If $a^m = a^n$
then, $m = n$

♦ If $a^m = b^m$
then, $a = b$

Checking Answers ✓

You can check the answer by substituting the value of x into the original equation.

$3^x \times 9^{x+5} \div 3^4 = 1$

Left   Right

Substitute $x = -2$ into left side of the equation

$3^{-2} \times 9^{-2+5} \div 3^4$

$= 3^{-2} \times 9^3 \div 3^4$

$= 3^{-2} \times 3^{2(3)} \div 3^4$

$= 3^{-2+6-4}$

$= 3^0$

$= 1$ ← The same value as the value on the right side of the equation.

Implementing the strategy

$3^x \times 9^{x+5} \div 3^4 = 1$ $3x + 6 = 0$

$3^x \times 3^{2(x+5)} \div 3^4 = 3^0$ $3x = -6$

$3^{x+2(x+5)-4} = 3^0$ $x = \dfrac{-6}{3}$

$3^{x+2x+10-4} = 3^0$ $x = -2$

$3^{3x+6} = 3^0$

$a^m = a^n$
$m = n$

Making a conclusion

If $3^x \times 9^{x+5} \div 3^4 = 1$,
then, $x = -2$

Example 19

Calculate the possible values of x for the equation $3^{x^2} \times 3^{2x} = 3^{15}$.

Understanding the problem

Calculate the value of x which is part of the indices.

Planning a strategy

All the bases involved in the equation are the same.

Implementing the strategy

$3^{x^2} \times 3^{2x} = 3^{15}$ If $a^m = a^n$, then, $m = n$.

$3^{x^2 + 2x} = 3^{15}$

$x^2 + 2x = 15$ ← Solve the quadratic equation using factorisation method.

$x^2 + 2x - 15 = 0$

$(x - 3)(x + 5) = 0$

$x - 3 = 0$ or $x + 5 = 0$

$x = 0 + 3$ $x = 0 - 5$

$x = 3$ $x = -5$

Making a conclusion

The possible values of x for the equation $3^{x^2} \times 3^{2x} = 3^{15}$ are 3 and -5.

Checking Answers ✓

Substitute the values of x into the original equation.

$\underbrace{3^{x^2} \times 3^{2x}}_{\text{Left}} = \underbrace{3^{15}}_{\text{Right}}$

Substitute $x = 3$

Left: Right:
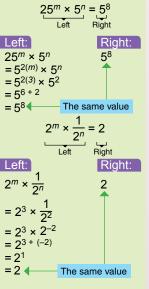
$3^{(3)^2} \times 3^{2(3)}$ 3^{15}

$= 3^9 \times 3^6$

$= 3^{9 + 6}$

$= 3^{15}$ ← The same value

Substitute $x = -5$

Left: Right:

$3^{(-5)^2} \times 3^{2(-5)}$ 3^{15}

$= 3^{25} \times 3^{-10}$

$= 3^{25 + (-10)}$

$= 3^{15}$ ← The same value

Example 20

Solve the following simultaneous equations.

$25^m \times 5^n = 5^8$ and $2^m \times \dfrac{1}{2^n} = 2$

Solution:

$25^m \times 5^n = 5^8$ $2^m \times \dfrac{1}{2^n} = 2$

$5^{2(m)} \times 5^n = 5^8$

$5^{2m + n} = 5^8$ $2^m \times 2^{-n} = 2^1$

$2m + n = 8 \rightarrow$ ① $2^{m + (-n)} = 2^1$

               $m - n = 1 \rightarrow$ ②

Equation ① and ② can be solved by substitution method.

From ① :

$2m + n = 8$

$n = 8 - 2m \rightarrow$ ③

Substitute ③ into ②

$m - n = 1$

$m - (8 - 2m) = 1$

$m - 8 + 2m = 1$

$m + 2m = 1 + 8$

$3m = 9$

$m = \dfrac{9}{3}$

$m = 3$

Substitute $m = 3$ into ①

$2m + n = 8$

$2(3) + n = 8$

$6 + n = 8$

$n = 8 - 6$

$n = 2$

> You can also substitute $m = 3$ into equation ② or ③.

Hence, $m = 3$ and $n = 2$.

🎬 FLASHBACK

Simultaneous linear equations in two variables can be solved using substitution method or elimination method.
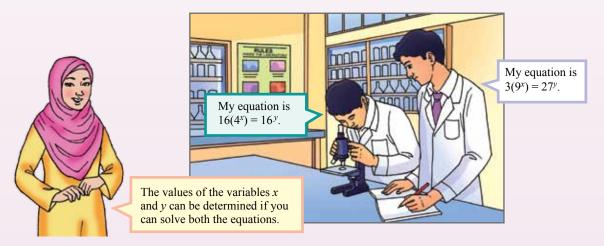
Checking Answers ✓

Substitute $m = 3$ and $n = 2$ into original simultaneous equations.

$\underbrace{25^m \times 5^n}_{\text{Left}} = \underbrace{5^8}_{\text{Right}}$

Left: Right:

$25^m \times 5^n$ 5^8

$= 5^{2(m)} \times 5^n$

$= 5^{2(3)} \times 5^2$

$= 5^{6 + 2}$

$= 5^8$ ← The same value

$\underbrace{2^m \times \dfrac{1}{2^n}}_{\text{Left}} = \underbrace{2}_{\text{Right}}$

Left: Right:

$2^m \times \dfrac{1}{2^n}$ 2

$= 2^3 \times \dfrac{1}{2^2}$

$= 2^3 \times 2^{-2}$

$= 2^{3 + (-2)}$

$= 2^1$

$= 2$ ← The same value

Example 21

My equation is $16(4^x) = 16^y$.

My equation is $3(9^x) = 27^y$.

The values of the variables x and y can be determined if you can solve both the equations.

Chong and Navin performed an experiment to determine the relationship between variable x and variable y. The equation Chong obtained was $16(4^x) = 16^y$, while the equation Navin got was $3(9^x) = 27^y$ as the findings of the experiment they performed. Calculate the value of x and of y which satisfy both the experiments Chong and Navin have performed.

Solution:

$$16(4^x) = 16^y \qquad 3(9^x) = 27^y$$
$$4^2(4^x) = 4^{2(y)} \qquad 3(3^{2x}) = 3^{3(y)}$$
$$4^{2+x} = 4^{2y} \qquad 3^{1+2x} = 3^{3y}$$
$$2 + x = 2y \rightarrow ① \qquad 1 + 2x = 3y \rightarrow ②$$

> You can also substitute $y = 3$ into equation ② or ③.

Equations ① and ② can be solved by elimination method.

> Multiply equation ① by 2 to equate the coefficients of variable x.

$$① \times 2 : 4 + 2x = 4y \rightarrow ③$$
$$② \qquad : 1 + 2x = 3y$$
$$③ - ② :$$
$$3 + 0 = y$$
$$y = 3$$

Substitute $y = 3$ into equation ①

$$① : 2 + x = 2y$$
$$2 + x = 2(3)$$
$$x = 6 - 2$$
$$x = 4$$

Hence, $x = 4$, $y = 3$
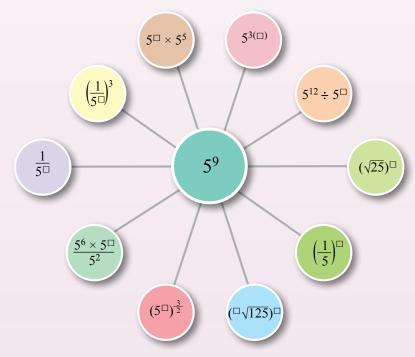
Dynamic Challenge

Test Yourself
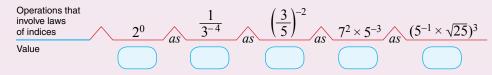
1. State whether each of the following operations which involves the laws of indices is **true** or **false**. If it is false, state the correct answer.

   (a) $a^5 = a \times a \times a \times a \times a$

   (b) $5^2 = 10$

   (c) $3^0 = 0$

   (d) $(2x^3)^5 = 2x^{15}$

   (e) $m^0 n^0 = 1$

   (f) $2a^{-4} = \dfrac{1}{2a^4}$

   (g) $32^{\frac{2}{5}} = (^2\sqrt{32})^5$

   (h) $\left(\dfrac{m}{n}\right)^{-4} = \left(\dfrac{n}{m}\right)^4$

   (i) $(5m^{\frac{1}{4}})^{-4} = \dfrac{625}{m}$

2. Copy and complete the following diagram with suitable values.

Circles around center 5^9:
- $5^{\square} \times 5^5$
- $5^{3(\square)}$
- $\left(\dfrac{1}{5^{\square}}\right)^3$
- $5^{12} \div 5^{\square}$
- $\dfrac{1}{5^{\square}}$
- $(\sqrt{25})^{\square}$
- $\dfrac{5^6 \times 5^{\square}}{5^2}$
- $\left(\dfrac{1}{5}\right)^{\square}$
- $(5^{\square})^{\frac{3}{2}}$
- $(\square\sqrt{125})^{\square}$

3. Copy and complete the following diagram.

Operations that involve laws of indices | 2^0 | $\dfrac{1}{3^{-4}}$ | $\left(\dfrac{3}{5}\right)^{-2}$ | $7^2 \times 5^{-3}$ | $(5^{-1} \times \sqrt{25})^3$

as ... as ... as ... as ...

Value | ☐ | ☐ | ☐ | ☐ | ☐

Skills Enhancement

1. Simplify each of the following.

(a) $(mn^4)^3 \div m^4 n^5$

(b) $3x \times \dfrac{1}{6} y^4 \times (xy)^3$

(c) $\sqrt{xy} \times \sqrt[3]{xy^2} \times \sqrt[6]{xy^5}$

2. Calculate the value of each of the following.

(a) $64^{\frac{1}{3}} \times 5^{-3}$

(b) $7^{-1} \times 125^{\frac{2}{3}}$

(c) $(256)^{\frac{3}{8}} \times 2^{-3}$

(d) $2^4 \times 16^{-\frac{3}{4}}$

(e) $\sqrt{49} \times 3^{-2} \div (\sqrt{81})^{-1}$

(f) $(125)^{\frac{2}{3}} \times (25)^{-\frac{3}{2}} \div (625)^{-\frac{1}{4}}$

3. Calculate the value of x for each of the following equations.

(a) $2^6 \div 2^x = 8$

(b) $3^{-4} \times 81 = 3^x$

(c) $a^x a^8 = 1$

(d) $4 \times 8^{x+1} = 2^{2x}$

(e) $(a^x)^2 \times a^5 = a^{3x}$

(f) $2^x = \dfrac{2^{10}}{16^x}$

(g) $3^6 \div 3^x = 81^{(x-1)}$

(h) $(m^2)^x \times m^{(x+1)} = m^{-2}$

(i) $25^x \div 125 = \dfrac{1}{5^x}$

Self Mastery

1. Calculate the value of each of the following without using a calculator.

   (a) $4^{\frac{1}{3}} \times 50^{\frac{2}{3}} \times 10^{\frac{5}{3}}$

   (b) $5^{\frac{5}{2}} \times 20^{\frac{3}{2}} \div 10^{-2}$

   (c) $60^{\frac{1}{2}} \times 125^{\frac{2}{3}} \div \sqrt{15}$

2. Calculate the value of x for each of the following equations.

   (a) $64x^{\frac{1}{2}} = 27x^{-\frac{5}{2}}$

   (b) $3x^{\frac{2}{3}} = \dfrac{27}{4} x^{-\frac{4}{3}}$

   (c) $25x^{-\frac{2}{3}} - \dfrac{5}{3} x^{\frac{1}{3}} = 0$

3. Calculate the possible values of x for each of the following equations.

   (a) $a^{x^2} \div a^{5x} = a^6$

   (b) $2^{x^2} \times 2^{6x} = 2^7$
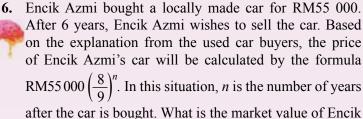
   (c) $5^{x^2} \div 5^{3x} = 625$

4. Solve the following simultaneous equations.

   (a) $81^{(x+1)} \times 9^x = 3^5$ and $8^{2x} \times 4(2^{2y}) = 128$

   (b) $4(4^x) = 8^{y+2}$ and $9^x \times 27^y = 1$

5. In an experiment performed by Susan, it was found that the temperature of a metal rose from 25°C to T°C according to equation $T = 25(1.2)^m$ when the metal was heated for m seconds. Calculate the difference in temperature between the fifth second and the sixth second, to the nearest degree Celsius.

6. Encik Azmi bought a locally made car for RM55 000. After 6 years, Encik Azmi wishes to sell the car. Based on the explanation from the used car buyers, the price of Encik Azmi's car will be calculated by the formula RM55 000 $\left(\dfrac{8}{9}\right)^n$. In this situation, n is the number of years after the car is bought. What is the market value of Encik Azmi's car? State your answer correct to the nearest RM.

7. Mrs Kiran Kaur saved RM50 000 on 1 March 2019 in a local bank with an interest of 3.5% per annum. After t years, Mrs Kiran Kaur's total savings, in RM, is 50 000 $(1.035)^t$. Calculate her total savings on 1 March 2025, if Mrs Kiran Kaur does not withdraw her savings.
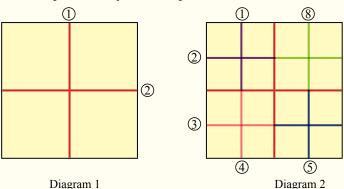
CHAPTER **1**

PROJECT 📋

Materials: One sheet of A4 paper, a pair of scissors, a long ruler, a pencil.

Instructions: (a) Carry out the project in small groups.
(b) Cut the A4 paper into the shape of a square. (Biggest possible)

Steps:

1. Draw the axes of symmetry (vertical and horizontal only) as shown in Diagram 1.

2. Calculate the number of squares formed. Write your answers in the space provided in Sheet A.

3. Draw the vertical and horizontal axes of symmetry for each square as shown in Diagram 2.

4. Calculate the number of squares formed. Write your answers in the space provided in Sheet A.

5. Repeat step 3 and step 4 as many times as possible.

Diagram 1

Diagram 2

6. Compare your answers with those of other groups.

7. What can you say about the patterns in the column 'Index form' in Sheet A?

8. Discuss the patterns you identify.

Scan the QR Code or visit http://bukutekskssm.my/Mathematics/F3/Chapter1SheetA.pdf to download Sheet A.

Sheet A

Number of axes of symmetry	Index form	Number of squares	Index form
0	–	1	2^0
2	2^1	4	2^2
8		16	

CONCEPT MAP

Indices

a^n ←——Index

a ↑——Base

$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}}$

$5^4 = 5 \times 5 \times 5 \times 5$

$m \times m \times m \times m \times m = m^5$

Multiplication

$a^m \times a^n = a^{m+n}$

$2^3 \times 2^5 = 2^{3+5}$

Division

$a^m \div a^n = a^{m-n}$

$3^6 \div 3^4 = 3^{6-4}$

Power

$(a^m)^n = a^{mn}$ $(a^m \times a^n)^p = a^{mp} \times a^{np}$

$(3^4)^2 = 3^8$ $(3a^4)^3 = 27a^{12}$

Fractional index

$a^{\frac{1}{n}} = \sqrt[n]{a}$

$a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = (a^{\frac{1}{n}})^m$

$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

$8^{\frac{1}{3}} = \sqrt[3]{8}$

$8^{\frac{2}{3}} = (8^2)^{\frac{1}{3}} = (8^{\frac{1}{3}})^2$

$8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2$

Negative index

$a^{-n} = \dfrac{1}{a^n} \; ; \; a \neq 0$

$5^{-3} = \dfrac{1}{5^3}$

Zero index

$a^0 = 1 \; ; \; a \neq 0$

$2^0 = 1$

$m^0 = 1$

SELF-REFLECT

At the end of this chapter, I can:

|		😊	🙁
1.	Represent repeated multiplication in index form and describe its meaning.		
2.	Rewrite a number in index form and vice versa.		
3.	Relate the multiplication of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.		
4.	Relate the division of numbers in index form with the same base, to repeated multiplications, and hence make generalisation.		
5.	Relate the numbers in index form raised to a power, to repeated multiplication, and hence make generalisation.		
6.	Verify that $a^0 = 1$ and $a^{-n} = \dfrac{1}{a^n} \; ; \; a \neq 0$.		
7.	Determine and state the relationship between fractional indices and roots and powers.		
8.	Perform operations involving laws of indices.		
9.	Solve problems involving laws of indices.		
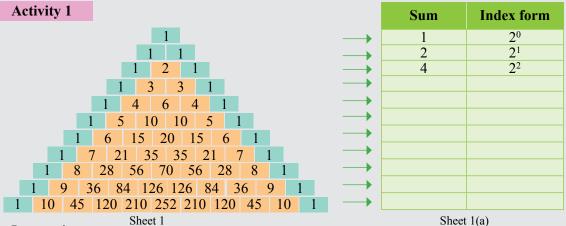
⚙ **EXPLORING MATHEMATICS**
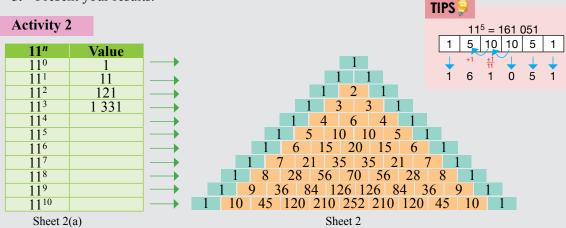
Do you still remember the Pascal's Triangle that you learnt in the Chapter 1 Patterns and Sequences in Form 2?

The Pascal's Triangle, invented by a French mathematician, Blaise Pascal, has a lot of unique properties. Let us explore two unique properties found in the Pascal's Triangle.

Activity 1

Sheet 1

Sum	Index form
1	2^0
2	2^1
4	2^2

Sheet 1(a)

Instructions:

1. Carry out the activity in pairs.
2. Construct the Pascal's Triangle as in Sheet 1.
3. Calculate the sum of the numbers in each row. Write the sum in index form with base of 2.
4. Complete Sheet 1(a). Discuss with your friends about the patterns of answers obtained.
5. Present your results.

Activity 2

11^n	Value
11^0	1
11^1	11
11^2	121
11^3	1 331
11^4	
11^5	
11^6	
11^7	
11^8	
11^9	
11^{10}	

Sheet 2(a)

Sheet 2

TIPS 💡

$11^5 = 161\ 051$

| 1 | 5 | 10 | 10 | 5 | 1 |

| 1 | 6 | 1 | 0 | 5 | 1 |

Instructions:

1. Carry out the activity in small groups.
2. Construct the Pascal's Triangle as in Sheet 2.
3. Take note on the numbers in each row. Each number is the value of index with base of 11.
4. Complete Sheet 2(a) with the value of index with base of 11 without using a calculator.
5. Present your results.
6. Are your answers the same as those of other groups?

CHAPTER 2

Standard Form

What will you learn?
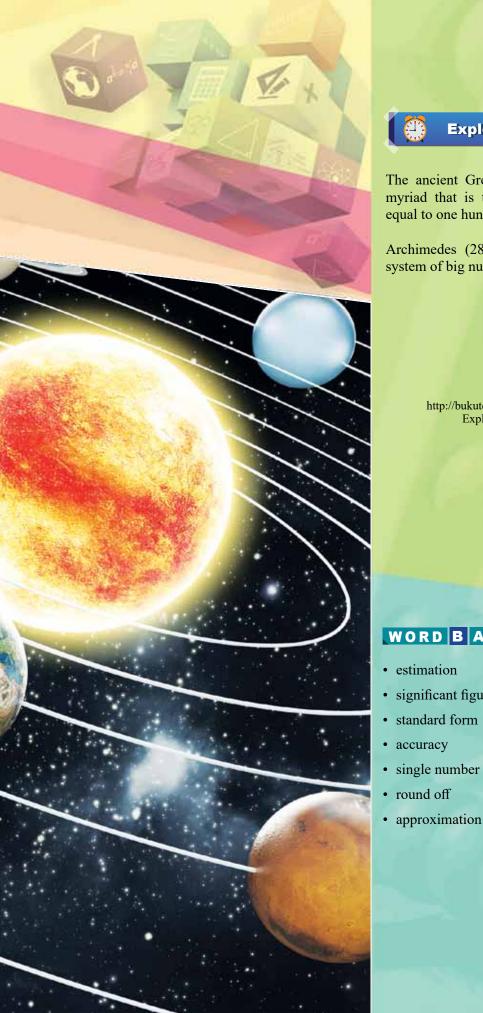
2.1 Significant Figures

2.2 Standard Form

Why do you learn this chapter?

- In scientific information, very big or very small numbers are commonly used. For example in astronomy, the distance between two stars is usually millions of kilometres while in the study of particles, the distance between atoms is extremely small.
- Numbers written in standard form are widely used in the field of science, engineering, astronomy and so on.

Distance in outer space, such as the distance between two stars in the galaxy, is measured in light years. One light year is the distance travelled by light in one year. One light year is equal to 9 500 000 000 000 km, that is 9.5 trillion kilometres. Small units such as nanometre are used for distances closer to zero. Do you know that 1 nanometre is equal to 0.000 000 001 metre?

The ancient Greeks used a system based on myriad that is ten thousand. Ten myriads is equal to one hundred thousand.

Archimedes (287 BC – 212 BC) created a system of big numbers up to $10^8 \times 10^{16}$.

WORD BANK

- estimation
- significant figure
- standard form
- accuracy
- single number
- round off
- approximation

- *anggaran*
- *angka bererti*
- *bentuk piawai*
- *kejituan*
- *nombor tunggal*
- *pembundaran*
- *penghampiran*

2.1 Significant Figures

✏️ **What does significant figure mean and how do you determine the number of significant figures of a number?**

We use measurement in many situations in our daily life. Examples of frequently used measurements are length, distance, mass, temperature, area and speed.

384 400 km

Earth

Moon

The **estimation** of a measurement can be done using **approximation**. For example, the distance between the Earth and the Moon is 384 400 km. This value is an estimation calculated using certain methods and stated as an approximation.

The **degree of approximation** of a measurement to the **actual value** shows the level of accuracy of the measurement. The skill in making estimations and approximations can help you in many situations in daily life.

Brainstorming 1 🔭 👥 In pairs

Aim: Determine the importance of making estimations and approximations in daily life.

Steps:

1. Read and understand the situations below.

> **Situation 1**
> Hashim is interested in a shirt sold in a supermarket with a 50% discount. The original price of the shirt is RM47.90. Hashim estimates the price of the shirt after discount and takes it to the cashier. The cashier informs him that the price of the shirt is RM28.70. Hashim argues that his estimation of the price is not more than RM25. Is Hashim's estimation correct?

50% DISCOUNT

> **Situation 2**
> Mrs Tan wants to buy 30 metres of cloth costing RM5.85 per metre to make curtains. She makes an estimation of the total price of the cloth and allocates RM180. Is the money allocated by Mrs Tan sufficient?

Discussion:

1. In the two situations above, how did Hashim and Mrs Tan make estimations of the total price?
2. Discuss with your friend the importance of making estimations and approximations.
3. State two other situations that require you to make estimations and approximations.

From Brainstorming 1, it is found that;

> Approximating a value to a certain significant figure allows us to make an accurate estimation.

You have understood the importance of making estimation for the purpose of obtaining a value that is near the exact value. Significant figures are used to obtain the approximate value.

> The significant figures of an integer or decimal refer to the digits in the number state accurately to a certain degree of accuracy as required. The number of significant numbers is counted starting from a non-zero digit.

Brainstorming 2

In pairs

Aim: Determine the effect of the position of the zero digit in integers and decimals.

Steps:

1. Study the integer cards below.

3 210	3 201	3 021	0 321
Card 1	**Card 2**	**Card 3**	**Card 4**

   Does the position of the zero digit have any effect on the value of digit 3?

2. Study the decimal cards below.

3.210	3.201	3.021	0.321
Card 5	**Card 6**	**Card 7**	**Card 8**

   Does the position of the zero digit have any effect on the value of digit 3?

3. Study the decimal cards below.

3.210	3.2100	3.21000	3.210000
Card 9	**Card 10**	**Card 11**	**Card 12**

   Does the position of the zero digit have any effect on the value of digit 2?

4. Discuss with your friend the effect of the position of the zero digit on the value of digit 3 in Card 1 to Card 8 and the effect of adding zero digits on the value of digit 2 in Card 9 to Card 12.

5. Present the results of your discussion. Compare your results with other pairs.

Discussion:

What is your conclusion concerning the position of the zero digit in an integer or decimal?

From Brainstorming 2, it is found that;

(a) Card 1, Card 2, Card 3, Card 5, Card 6 and Card 7
   • The position of the zero digit between or at the end of the number, maintains the place value of digit 3.
(b) Card 4 and Card 8
   • The position of the zero digit as the first digit has changed the place value of digit 3.
(c) Card 9, Card 10, Card 11 and Card 12
   • The position of the zero digit at the end of the decimal does not change the place value of digit 2.

FLASHBACK

For digit 9 in the number 5 9 2 7;
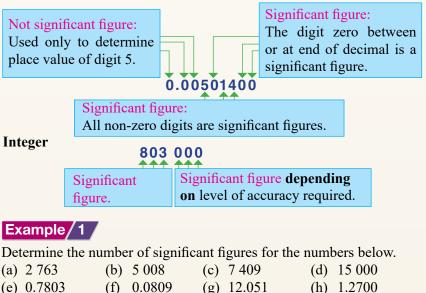♦ Place value – hundred
♦ Digit value – 900

In general,

- All non-zero digits are significant figures.
- The digit zero between non-zero digits is a significant figure.
- The digit zero at the end of an integer is a significant figure according to the level of accuracy required.
- The digit zero at the end of a decimal is a significant figure because it determines the level of accuracy of the decimal.
- The digit zero before the first non-zero digit is not a significant figure.

How do you determine the number of significant figures?

Decimal

| Not significant figure: Used only to determine place value of digit 5. | Significant figure: The digit zero between or at end of decimal is a significant figure. |

0.00501400

| Significant figure: All non-zero digits are significant figures. |

Integer

803 000

| Significant figure. | Significant figure **depending on** level of accuracy required. |

Example 1

Determine the number of significant figures for the numbers below.
(a) 2 763   (b) 5 008   (c) 7 409   (d) 15 000
(e) 0.7803   (f) 0.0809   (g) 12.051   (h) 1.2700

Solution:

(a) 2 763 [4 s.f.]

(b) 5 008 [4 s.f.] ⟶ The digit zero between non-zero digit is a significant figure.
(c) 7 409 [4 s.f.] ⟶ The digit zero between non-zero digit is a significant figure.
(d) (i)   15 000 [2 s.f.] ⟶ If level of accuracy is to the nearest thousand.
    (ii) 15 000 [3 s.f.] ⟶ If level of accuracy is to the nearest hundred.
    (iii) 15 000 [4 s.f.] ⟶ If level of accuracy is to the nearest ten.
    (iv) 15 000 [5 s.f.] ⟶ If level of accuracy is to the nearest one.
(e) 0.7803 [4 s.f.] ⎫
(f) 0.0809 [3 s.f.] ⎭ ⟶ The digit zero before first non-zero digit is not significant figure.
(g) 12.051 [5 s.f.]
(h) 1.2700 [5 s.f.] ⟶ All zeros after non-zero digit at end of decimal are significant figures.

MIND TEST 2.1a

1. State the number of significant figures for the following numbers.
   (a) 2 600   (b) 30 004   (c) 4 000 600   (d) 0.5003
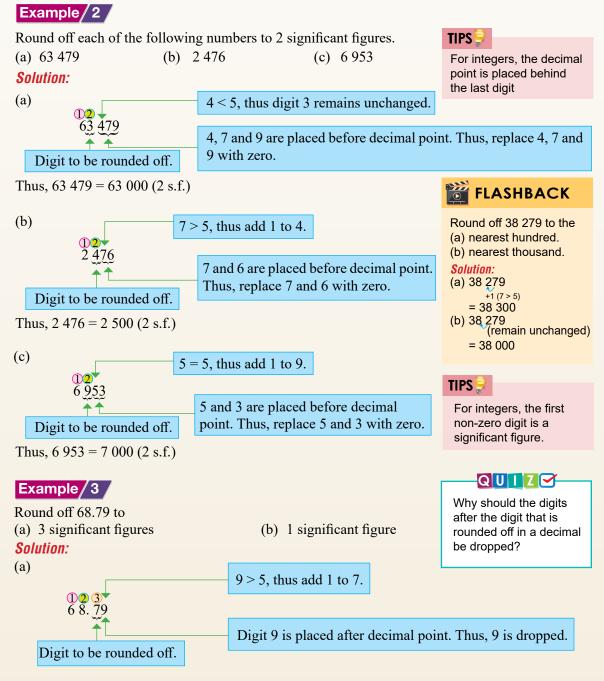   (e) 0.080   (f) 9.0070   (g) 0.002000   (h) 30.0002

✎ How do you round off a number to certain numbers of significant figures?

Do you still remember how to round off a number to a certain place value? The same concept and method are used to round off a number to a certain number of significant figures.
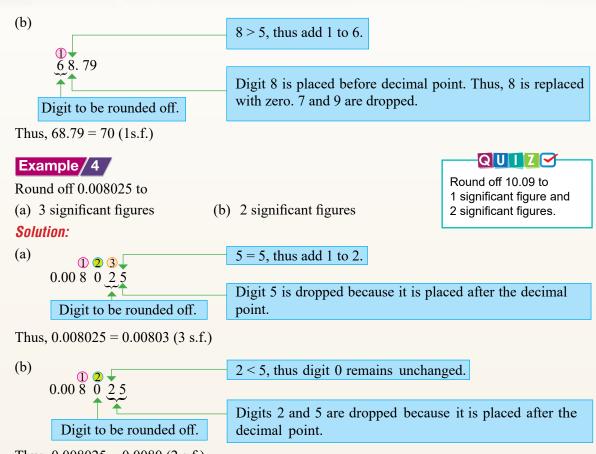

Example 2

Round off each of the following numbers to 2 significant figures.
(a) 63 479　　　　　　　(b) 2 476　　　　　　　(c) 6 953

Solution:

(a)
① ②
63 479

4 < 5, thus digit 3 remains unchanged.

Digit to be rounded off.

4, 7 and 9 are placed before decimal point. Thus, replace 4, 7 and 9 with zero.

Thus, 63 479 = 63 000 (2 s.f.)

(b)
① ②
2 476

7 > 5, thus add 1 to 4.

Digit to be rounded off.

7 and 6 are placed before decimal point. Thus, replace 7 and 6 with zero.

Thus, 2 476 = 2 500 (2 s.f.)

(c)
① ②
6 953

5 = 5, thus add 1 to 9.

Digit to be rounded off.

5 and 3 are placed before decimal point. Thus, replace 5 and 3 with zero.

Thus, 6 953 = 7 000 (2 s.f.)

TIPS

For integers, the decimal point is placed behind the last digit

FLASHBACK

Round off 38 279 to the
(a) nearest hundred.
(b) nearest thousand.
Solution:
(a) 38 279
　　$+1 (7 > 5)$
　= 38 300
(b) 38 279
　　(remain unchanged)
　= 38 000

TIPS

For integers, the first non-zero digit is a significant figure.

QUIZ

Why should the digits after the digit that is rounded off in a decimal be dropped?

Example 3

Round off 68.79 to
(a) 3 significant figures　　　　　　　(b) 1 significant figure

Solution:

(a)
① ② ③
6 8. 79

9 > 5, thus add 1 to 7.

Digit to be rounded off.

Digit 9 is placed after decimal point. Thus, 9 is dropped.

Thus, 68.79 = 68.8 (3 s.f.)

CHAPTER 2

(b)

$$8 > 5, \text{ thus add 1 to 6.}$$

①
6̲ 8. 79

Digit to be rounded off.

Digit 8 is placed before decimal point. Thus, 8 is replaced with zero. 7 and 9 are dropped.

Thus, $68.79 = 70$ (1s.f.)

Example 4

Round off 0.008025 to

(a) 3 significant figures (b) 2 significant figures

QUIZ ✓

Round off 10.09 to
1 significant figure and
2 significant figures.

Solution:

(a)

① ② ③
0.00 8 0 2̲ 5

Digit to be rounded off.

$5 = 5$, thus add 1 to 2.

Digit 5 is dropped because it is placed after the decimal point.

Thus, $0.008025 = 0.00803$ (3 s.f.)

(b)

① ②
0.00 8 0 2̲ 5

Digit to be rounded off.

$2 < 5$, thus digit 0 remains unchanged.

Digits 2 and 5 are dropped because it is placed after the decimal point.

Thus, $0.008025 = 0.0080$ (2 s.f.)

MIND TEST 2.1b

1. Complete the table below by rounding off each number below to the given significant figure.

Number	3 significant figures	2 significant figures	1 significant figure
(a) 47 193			
(b) 5 261			
(c) 305.72			
(d) 20.68			
(e) 8.595			
(f) 5.9			
(g) 0.6937			
(h) 0.09184			
(i) 0.005709			

2. Calculate each operation below. State the answer to the significant figures shown in the brackets.

(a) $2.57 \times 4.5 + 0.45$ [4]

(b) $8.59 \div 2.1 - 1.26$ [3]

(c) $14.23 - 2.6 \times 1.2$ [3]

(d) $15.74 + 20.3 \div 2.5$ [2]

(e) $7.63 \times 0.5 \div 4.2 + 5.7$ [3]

(f) $10.25 \div 0.75 - 4.2 \times 0.2$ [2]

(g) $15.62 - 1.72 \times 0.2 + 6.3$ [1]

(h) $4.94 + 5.76 \div 0.26 \times 1.4$ [3]

2.2 Standard Form

✏️ How do you recognise and write numbers in standard form?

Many scientific fields such as astronomy, biology, physics and engineering frequently use numbers that are too big or too small in their research. These numbers are written in standard form to make writing easier.

Standard form is a way to write a **single number** in the form;

$$A \times 10^n$$

where $1 \leq A < 10$ and n is an integer.

For example, the land area of Malaysia is 330 803 000 000 m². This value can be written as 3.308×10^{11} m² or 3.30803×10^{11} m² or depending on the number of significant figures required.

LEARNING STANDARD

Recognise and write numbers in standard form.

🔢 How do you change a single number to standard form?

When a single number is changed to standard form;

- Numbers with value more than 1 will be given positive index.
- Numbers with value less than 1 will be given negative index.

FLASHBACK

♦ a^n, n is a positive index.
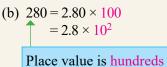♦ a^{-n}, $-n$ is a negative index.

Example 5

Write the following single numbers in standard form.
(a) 28
(b) 280
(c) 2 805.3

Solution:

(a) $28 = 2.8 \times 10$

Place value is tens

Decimal point after first non-zero digit.

(b) $280 = 2.80 \times 100$
     $= 2.8 \times 10^2$

Place value is hundreds

(c) $2\ 805.3 = 2.8053 \times 1\ 000$
     $= 2.8053 \times 10^3$

Place value is thousands

Example 6

Write the following decimals in standard form.
(a) 0.325
(b) 0.00325
(c) 0.03025
(d) 0.003005

Solution:

(a) $0.325 = 3.25 \times \dfrac{1}{10}$

     $= 3.25 \times 10^{-1}$

Place value is one tenths

(b) $0.00325 = 3.25 \times \dfrac{1}{1\ 000}$

     $= 3.25 \times \dfrac{1}{10^3}$

     $= 3.25 \times 10^{-3}$

Place value is one thousandths

FLASHBACK

$\dfrac{1}{a^n} = a^{-n}$

DISCUSSION CORNER 💬

Is 5.1×10^0 a number in standard form? Discuss.

(c) $0.03025 = 3.025 \times \dfrac{1}{100}$

$\qquad = 3.025 \times \dfrac{1}{10^2}$

$\qquad = 3.025 \times 10^{-2}$

Place value is one hundredths

(d) $0.003005 = 3.005 \times \dfrac{1}{1\,000}$

$\qquad = 3.005 \times \dfrac{1}{10^3}$

$\qquad = 3.005 \times 10^{-3}$

Place value is one thousandths

How do you change a number in standard form to single number?

When a number in standard form is changed to a single number;

- The number will be equal to 10 or more if the index is positive.
- The number will be less than 1 if the index is negative.

Example 7

Write 4.17×10^5 as a single number.

Solution:

$4.17 \times 10^5 = 4.17 \times 100\,000$

$\qquad\qquad = 417\,000$

Example 8

Write 8.063×10^{-5} as a single number.

Solution:

$8.063 \times 10^{-5} = 8.063 \times \dfrac{1}{100\,000}$

$\qquad\qquad = 0.00008063$

Example 9

Determine 3 050 terabytes in bytes. State the answer in standard form.

Solution:

$\begin{aligned}
3\,050 \text{ terabytes} &= 3\,050 \times 10^{12} \text{ bytes}\\
&= (3.05 \times 10^3) \times 10^{12} \text{ bytes}\\
&= (3.05 \times 10^{3+12}) \text{ bytes} \quad\leftarrow \text{Use index law } a^m \times a^n = a^{m+n}\\
&= 3.05 \times 10^{15} \text{ bytes}
\end{aligned}$

Example 10

Determine 0.0057 nanometre in metre. State your answer in standard form.

Solution:

$\begin{aligned}
0.0057 \text{ nanometre} &= 0.0057 \times 10^{-9} \text{ metre.}\\
&= (5.7 \times 10^{-3}) \times 10^{-9} \text{ metre}\\
&= (5.7 \times 10^{-3+(-9)}) \text{ metre} \quad\leftarrow \text{Use index law } a^m \times a^n = a^{m+n}\\
&= (5.7 \times 10^{-3-9}) \text{ metre}\\
&= 5.7 \times 10^{-12} \text{ metre}
\end{aligned}$

Brainstorming 3 👀

In pairs

Aim: Write metric measurements in standard form.

Steps:

1. Complete the table below by writing the single numbers for metric measurements in standard form.

Prefix	Symbol	Value	
		Single number	Standard form
exa	E	1 000 000 000 000 000 000	1×10^{18}
peta	P	1 000 000 000 000 000	
tera	T	1 000 000 000 000	
giga	G	1 000 000 000	
mega	M	1 000 000	
kilo	k	1 000	
hecto	h	100	
deca	da	10	
–	–	1	1×10^{0}
deci	d	0.1	1×10^{-1}
centi	c	0.01	
milli	m	0.001	
micro	μ	0.000 001	
nano	n	0.000 000 001	
pico	p	0.000 000 000 001	
femto	f	0.000 000 000 000 001	
atto	a	0.000 000 000 000 000 001	

Discussion:

A number with too big or too small value can be written as a single number or in standard form. Which form will you choose for an arithmetic operation? Give your reasons.

From Brainstorming 3, it is found that;

> Standard form makes it easier to write very big and very small numbers in a form that is simple and easy to understand. 🖋️

MIND TEST 2.2a

TIPS 💡
Use data from Brainstorming 3 to solve question 3.

1. Write the following single numbers in standard form.
   (a) 35 (b) 481 (c) 5 075 (d) 97.25
   (e) 3 124.3 (f) 0.9 (g) 0.23 (h) 0.0375

2. Change the numbers in standard form to single numbers.
   (a) 2.5×10^{0} (b) 3.75×10^{1} (c) 4.23×10^{2}
   (d) 5.07×10^{3} (e) 9.1×10^{4} (f) 6.2×10^{-1}
   (g) 7.29×10^{-2} (h) 1.034×10^{-3} (i) 8.504×10^{-4}

3. Change the following metric measurements to the units given in the brackets. State your answers in standard form.
   (a) 1 050 kilometres [metre] (b) 216 gigabytes [byte]
   (c) 0.75 teralitre [litre] (d) 95 micrometres [metre]
   (e) 123 nanometres [metre] (f) 0.089 femtometre [metre]

How are basic arithmetic operations involving numbers in standard form performed?

Add and subtract operations

Example 11

Calculate the value of each of the following operations. State your answer in standard form.

(a) $2.73 \times 10^3 + 5.92 \times 10^3$

(b) $4.27 \times 10^5 + 9.35 \times 10^5$

(c) $7.02 \times 10^4 + 2.17 \times 10^5$

(d) $9.45 \times 10^6 - 3.24 \times 10^5$

Solution:

(a) $2.73 \times 10^3 + 5.92 \times 10^3$

$= (2.73 + 5.92) \times 10^3$

$= 8.65 \times 10^3$

Factorise 10^3

(b) $4.27 \times 10^5 + 9.35 \times 10^5$

$= (4.27 + 9.35) \times 10^5$

$= 13.62 \times 10^5$

$= (1.362 \times 10) \times 10^5$

$= 1.362 \times 10^1 \times 10^5$

$= 1.362 \times 10^{1+5}$

$= 1.362 \times 10^6$

FLASHBACK

- $5a^n + 7a^n$
  $= (5 + 7)a^n$
  $= 12a^n$

- $5 \times 10^n + 7 \times 10^n$
  $= (5 + 7)10^n$
  $= 12(10^n)$

(c)

Method 1

$7.02 \times 10^4 + 2.17 \times 10^5$

$= 7.02 \times 10^4 + 2.17 \times 10^1 \times 10^4$

$= 7.02 \times 10^4 + 21.7 \times 10^4$

$= (7.02 + 21.7) \times 10^4$

$= 28.72 \times 10^4$

$= 2.872 \times 10^1 \times 10^4$

$= 2.872 \times 10^{1+4}$

$= 2.872 \times 10^5$

Method 2

$7.02 \times 10^4 + 2.17 \times 10^5$

$= 7.02 \times 10^{-1} \times 10^5 + 2.17 \times 10^5$

$= 0.702 \times 10^5 + 2.17 \times 10^5$

$= (0.702 + 2.17) \times 10^5$

$= 2.872 \times 10^5$

10^5 change to $10^1 \times 10^4$ to simply calculation.

TIPS

For operations of addition and subtraction, change index with small value to index with large value as in method 2 of example (c) and example (d).

(d)

Method 1

$9.45 \times 10^6 - 3.24 \times 10^5$

$= 9.45 \times 10^1 \times 10^5 - 3.24 \times 10^5$

$= 94.5 \times 10^5 - 3.24 \times 10^5$

$= (94.5 - 3.24) \times 10^5$

$= 91.26 \times 10^5$

$= 9.126 \times 10^1 \times 10^5$

$= 9.126 \times 10^{1+5}$

$= 9.126 \times 10^6$

Method 2

$9.45 \times 10^6 - 3.24 \times 10^5$

$= 9.45 \times 10^6 - 3.24 \times 10^{-1} \times 10^6$

$= 9.45 \times 10^6 - 0.324 \times 10^6$

$= (9.45 - 0.324) \times 10^6$

$= 9.126 \times 10^6$

SMART MIND

Calculate the following without using a calculator.

- $2.4 \times 10^3 + 1.3 \times 10^5$
- $8.5 \times 10^4 - 1.2 \times 10^2$

Example 12

Calculate the value of each of the following operations. State the answer in standard form.

(a) $3.58 \times 10^{-3} + 9.24 \times 10^{-3}$ (b) $8.21 \times 10^{-4} + 1.49 \times 10^{-5}$
(c) $2.3 \times 10^{-5} - 4.6 \times 10^{-6}$

Solution:
(a) $3.58 \times 10^{-3} + 9.24 \times 10^{-3} = (3.58 + 9.24) \times 10^{-3}$
$$= 12.82 \times 10^{-3}$$
$$= 1.282 \times 10^{1} \times 10^{-3}$$
$$= 1.282 \times 10^{1 + (-3)}$$
$$= 1.282 \times 10^{-2}$$

(b)

Method 1

$8.21 \times \boxed{10^{-4}} + 1.49 \times 10^{-5}$
$= 8.21 \times \boxed{10^{1} \times 10^{-5}} + 1.49 \times 10^{-5}$
$= 82.1 \times 10^{-5} + 1.49 \times 10^{-5}$
$= (82.1 + 1.49) \times 10^{-5}$
$= 83.59 \times 10^{-5}$
$= 8.359 \times 10^{1} \times 10^{-5}$
$= 8.359 \times 10^{1 + (-5)}$
$= 8.359 \times 10^{-4}$

Method 2

$8.21 \times 10^{-4} + 1.49 \times \boxed{10^{-5}}$
$= 8.21 \times 10^{-4} + 1.49 \times \boxed{10^{-1} \times 10^{-4}}$
$= 8.21 \times 10^{-4} + 0.149 \times 10^{-4}$
$= (8.21 + 0.149) \times 10^{-4}$
$= 8.359 \times 10^{-4}$

(c)

Method 1

$2.3 \times \boxed{10^{-5}} - 4.6 \times 10^{-6}$
$= 2.3 \times \boxed{10^{1} \times 10^{-6}} - 4.6 \times 10^{-6}$
$= 23 \times 10^{-6} - 4.6 \times 10^{-6}$
$= (23 - 4.6) \times 10^{-6}$
$= 18.4 \times 10^{-6}$
$= 1.84 \times 10^{1} \times 10^{-6}$
$= 1.84 \times 10^{1 + (-6)}$
$= 1.84 \times 10^{-5}$

Method 2

$2.3 \times 10^{-5} - 4.6 \times \boxed{10^{-6}}$
$= 2.3 \times 10^{-5} - 4.6 \times \boxed{10^{-1} \times 10^{-5}}$
$= 2.3 \times 10^{-5} - 0.46 \times 10^{-5}$
$= (2.3 - 0.46) \times 10^{-5}$
$= 1.84 \times 10^{-5}$

MIND TEST 2.2b

1. Calculate the value of each of the following operations. State your answer in standard form.

(a) $2.4 \times 10^{4} + 3.57 \times 10^{4}$ (b) $8.2 \times 10^{6} - 4.27 \times 10^{6}$
(c) $5.23 \times 10^{7} + 4.98 \times 10^{7}$ (d) $1.2 \times 10^{5} + 3.74 \times 10^{4}$
(e) $5.7 \times 10^{8} - 2.4 \times 10^{7}$ (f) $5.7 \times 10^{3} + 8.02 \times 10^{4}$
(g) $6.5 \times 10^{4} - 7.3 \times 10^{3}$ (h) $5.2 \times 10^{-3} - 4.12 \times 10^{-3}$
(i) $8.74 \times 10^{-5} - 2.65 \times 10^{-5}$ (j) $4.1 \times 10^{-4} + 9.5 \times 10^{-3}$
(k) $8.3 \times 10^{-4} - 6.2 \times 10^{-5}$ (l) $9.42 \times 10^{-6} - 7.35 \times 10^{-7}$

SMART TECHNOLOGY

1. Press **Mode** button a few times until the screen shows:
   Fix **Sci** **Norm**
   1 **2** **3**

2. Press **2** to choose **Sci**, that is standard form.

3. Enter number of significant figures (s.f.) needed, for example 9.

4. Enter the required operation.
   ♦ $3.2 \times 10^{5} - 4.2 \times 10^{4}$
   Press 3.2 **Exp** 5 − 4.2 **Exp** 4.

   Screen display:
   3.2 E5 − 4.2 E4
   Press **=** 2.78×10^{5}.

   ♦ $4 \times 10^{5} \times 3.7 \times 10^{4}$
   Press 4 **Exp** 5 × 3.7 **Exp** 4.

   Screen display:
   4 **Exp** 5 × 3.7 **Exp** 4
   Press **=** 1.48×10^{10}.

5. Extend your exploration to other operations involving other standard forms

6. Compare the results produced by calculator and answers obtained through manual calculations.

🔢 Operations of Multiplication and Division

TIPS 💡

Law of Indices
- Operation of multiplication
$(A \times 10^m) \times (B \times 10^n)$
$= (A \times B) \times 10^{m+n}$
- Operation of division
$(A \times 10^m) \div (B \times 10^n)$
$= (A \div B) \times 10^{m-n}$

Example 13

Solve the following operations. State your answers in standard form.

(a) $3 \times 10^5 \times 4.9 \times 10^2$

(b) $7.5 \times 10^{-3} \times 5 \times 10^{-6}$

(c) $\dfrac{5.9 \times 10^5}{2 \times 10^2}$

(d) $\dfrac{6.8 \times 10^{-3}}{4 \times 10^{-6}}$

Solution:

(a) $3 \times 10^5 \times 4.9 \times 10^2$
$= (3 \times 4.9) \times 10^{5+2}$
$= 14.7 \times 10^7$
$= 1.47 \times 10^1 \times 10^7$
$= 1.47 \times 10^{1+7}$
$= 1.47 \times 10^8$

(b) $7.5 \times 10^{-3} \times 5 \times 10^{-6}$
$= (7.5 \times 5) \times 10^{-3+(-6)}$
$= 37.5 \times 10^{-9}$
$= 3.75 \times 10^1 \times 10^{-9}$
$= 3.75 \times 10^{1+(-9)}$
$= 3.75 \times 10^{-8}$

(c) $\dfrac{5.9 \times 10^5}{2 \times 10^2}$
$= \dfrac{5.9}{2} \times 10^{5-2}$
$= 2.95 \times 10^3$

(d) $\dfrac{6.8 \times 10^{-3}}{4 \times 10^{-6}}$
$= \dfrac{6.8}{4} \times 10^{-3-(-6)}$
$= 1.7 \times 10^3$

MIND TEST 2.2c

1. Calculate the value of each of the following operations. State your answer in standard form.

   (a) $4 \times 10^5 \times 3.7 \times 10^2$

   (b) $7.5 \times 10^{-3} \times 5 \times 10^{-6}$

   (c) $6.3 \times 10^5 \times 4.0 \times 10^2$

   (d) $5.3 \times 10^{-3} \times 4 \times 10^5$

   (e) $(1.08 \times 10^2) \div (2.4 \times 10^4)$

   (f) $(9.6 \times 10^{-2}) \div (1.5 \times 10^{-5})$

   (g) $(5.9 \times 10^5) \div (2 \times 10^2)$

   (h) $(2.58 \times 10^4) \div (0.3 \times 10^{-4})$

2. A mobile swimming pool measures 305 cm × 183 cm × 56 cm. Calculate the maximum volume of water that it can hold in litres. State your answer in standard form and correct to four significant figures.

BULLETIN 📢

1 litre = 1 000 cm³
1 litre = 0.001 m³

3. Syazwani wants to transfer 2 terabytes of data to pen drives with a capacity of 32 gigabytes. What is the minimum number of 32-gigabyte pen drives needed?

DISCUSSION CORNER 💬

Between operation of addition or subtraction and operation of multiplication or division involving standard form, which operation is easier? Why?

4. Given 1 millimetre $= 10^{-3}$ metre and 1 micrometre $= 10^{-6}$ metre. State 1 millimetre in micrometre.
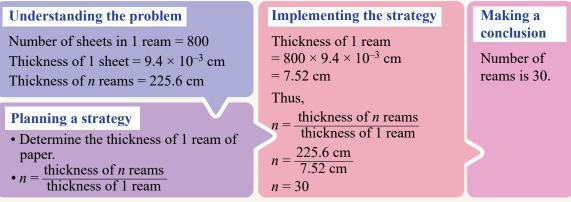
✎ **How do you solve problems involving numbers in standard form?**

Example 14

A ream of paper contains 800 sheets of paper. The thickness of one sheet of paper is 9.4×10^{-3} cm. Given the total thickness of n reams of paper is 225.6 em Calculate the value of n.

Solution:

Understanding the problem

Number of sheets in 1 ream $= 800$
Thickness of 1 sheet $= 9.4 \times 10^{-3}$ cm
Thickness of n reams $= 225.6$ cm

Planning a strategy

• Determine the thickness of 1 ream of paper.
• $n = \dfrac{\text{thickness of } n \text{ reams}}{\text{thickness of 1 ream}}$

Implementing the strategy

Thickness of 1 ream
$= 800 \times 9.4 \times 10^{-3}$ cm
$= 7.52$ cm
Thus,
$n = \dfrac{\text{thickness of } n \text{ reams}}{\text{thickness of 1 ream}}$
$n = \dfrac{225.6 \text{ cm}}{7.52 \text{ cm}}$
$n = 30$

Making a conclusion

Number of reams is 30.

Example 15

A property firm bought a piece of land in the shape of a right-angled triangle PQR as shown in the diagram.

(a) Calculate the value of PQ, in metres and state your answer in standard form.

(b) If the cost of one square metre of the land is RM45, calculate the total cost of the land in RM.

Solution:

Understanding the problem

$\triangle PQR$ is a right-angled triangle. QR is the hypotenuse.

Planning a strategy

(a) Calculate PQ using Pythagoras' theorem.
(b) Calculate the area of land in the shape of $\triangle PQR$. Multiply total land area by cost of 1 m² of land.

Implementing the strategy

(a) $PQ^2 = [(3.5 \times 10^2)^2 - (2.1 \times 10^2)^2]$ m²
$= [1.225 \times 10^5 - 4.41 \times 10^4]$ m²
$= (7.84 \times 10^4)$ m²
$PQ = \sqrt{(7.84 \times 10^4)}$ m²
$= 2.8 \times 10^2$ m

(b) Area of $\triangle PQR = \dfrac{1}{2} \times \left(2.1 \times 10^2\right)m\times \left(2.8 \times 10^2\right)$m
$= 2.94 \times 10^4$ m²
Cost of land $= 2.94 \times 10^4 \times$ RM45
$=$ RM1 323 000.00
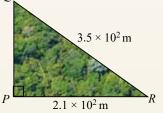
Making a conclusion

(a) Distance $PQ = 2.8 \times 10^2$ m
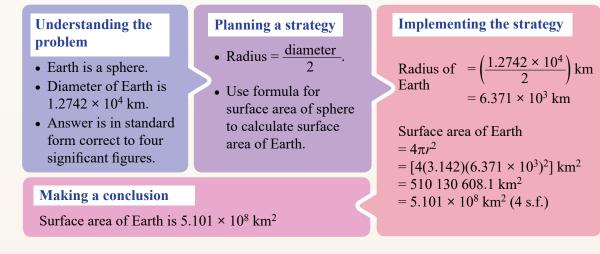
(b) Total cost of land $=$ RM1 323 000.00
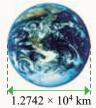
Example 16

The picture shows the Earth with a diameter of 1.2742×10^4 km. Calculate the surface area of the Earth, in km^2. State the answer in standard form correct to four significant figures. [Surface area of sphere $= 4\pi r^2$ and $\pi = 3.142$]
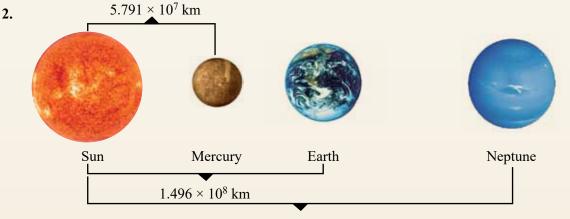
1.2742×10^4 km

Solution:

Understanding the problem

- Earth is a sphere.
- Diameter of Earth is 1.2742×10^4 km.
- Answer is in standard form correct to four significant figures.

Planning a strategy

- Radius $= \dfrac{diameter}{2}$.
- Use formula for surface area of sphere to calculate surface area of Earth.

Implementing the strategy

$$\text{Radius of Earth} = \left(\frac{1.2742 \times 10^4}{2}\right) \text{km}$$
$$= 6.371 \times 10^3 \text{ km}$$

Surface area of Earth
$$= 4\pi r^2$$
$$= [4(3.142)(6.371 \times 10^3)^2] \text{ km}^2$$
$$= 510\,130\,608.1 \text{ km}^2$$
$$= 5.101 \times 10^8 \text{ km}^2 \text{ (4 s.f.)}$$

Making a conclusion

Surface area of Earth is 5.101×10^8 km^2

MIND TEST 2.2d

1. The average daily water consumption in a residential area is $6\,950$ m^3. Calculate the total water consumption, in cubic metres, in the residential area for February 2016. State the answer in standard form correct to three significant figures.

2.

5.791×10^7 km

Sun   Mercury   Earth   Neptune

1.496×10^8 km

4.495×10^9 km

The picture above shows the estimated distance of three planets in the solar system from the Sun on a certain day. Calculate the difference in distance, in km, between

(a) Mercury and Earth   (b) Mercury and Neptune   (c) Earth and Neptune

State the answers in standard form correct to three significant figures.

Dynamic Challenge

Test Yourself

1. Round off the following numbers and decimals correct to the significant figures stated in the brackets.

   (a) 23 725 [2]   (b) 54 299 [4]   (c) 8 999 [2]   (d) 295 197 [2]

   (e) 4 854   [1]   (f) 5   [3]   (g) 0.2763 [2]   (h) 35.074 [1]

   (i) 423.575 [2]   (j) 10.234 [1]   (k) 1.0372 [3]   (l) 501.724 [3]

2. Given $m = 3.2 \times 10^3$ and $n = 5.43 \times 10^4$. Calculate the values of the following operations. State your answers in standard form correct to three significant figures.

   (a) $2mn$ (b) $m + n$ (c) $n - m$ (d) $m^2 + n^2$

   (e) $\dfrac{3m}{2n}$ (f) $\dfrac{m + n}{mn}$ (g) $m^{-2} + n^{-3}$ (h) $n - m^{-3}$

3. Complete the following.

   (a) $2.5 \times 10^2 + 1.35 \times 10^4$
   $= 2.5 \times 10^{\square} \times 10^4 + 1.35 \times 10^4$
   $= \boxed{} \times 10^4 + 1.35 \times 10^4$
   $= (\boxed{} + \boxed{}) \times 10^4$
   $= \boxed{} \times 10^4$

   (b) $5.74 \times 10^{-3} + 3.4 \times 10^{-6}$
   $= 5.74 \times 10^{-3} + 3.4 \times 10^{\square} \times 10$
   $= 5.74 \times 10^{-3} + \boxed{} \times 10^{-3}$
   $= (\boxed{} + \boxed{}) \times 10^{-3}$
   $= \boxed{} \times 10^{-3}$

   (c) $1.75 \times 10^2 - 4.2 \times 10^{-1}$
   $= 1.75 \times 10^2 - 4.2 \times 10^{\square} \times 10^2$
   $= 1.75 \times 10^2 - \boxed{} \times 10^2$
   $= (\boxed{} + \boxed{}) \times 10^2$
   $= \boxed{} \times 10^2$

   (d) $3.7 \times 10^{-2} - 4.3 \times 10^{-5}$
   $= 3.7 \times 10^{-2} - 4.3 \times 10^{\square} \times 10^{-2}$
   $= 3.7 \times 10^{-2} - \boxed{} \times 10^{-2}$
   $= (\boxed{} - \boxed{}) \times 10^{-2}$
   $= \boxed{} \times 10^{-2}$

4. A factory produces 72 thousand packets of chips every week. If the factory operates 6 days a week and 18 hours a day, calculate

   (a) the number of packets of chips produced every day. State your answer in standard form.

   (b) the average profit per hour if the net profit of one packet of chips is 32 sen. State the answer to the nearest RM.

CHAPTER 2

5. The estimated population of Malaysia for 2018 is 32 million. Given Malaysia's land area is 330 803 km², calculate the population density of Malaysia for each square kilometre for 2018.

State your answer correct to the nearest integer.

Skills Enhancement

1. A newly built community hall required 6 185 pieces of tiles measuring 30 cm × 30 cm for the floor.
   (a) Calculate the floor area of the hall in square metres. State your answer in standard form correct to three significant figures.
   (b) Given the cost of one piece of tile is RM1.75. Calculate the total cost of the tiles to the nearest RM.

2. Encik Hanif drove his car from Kota Bharu to Kuala Terengganu to visit his son. On the way back to Kota Bharu, Encik Hanif made a stop at Setiu. The map shows the distance and travelling time of Encik Hanif.
   (a) Calculate the average speed, in kmh⁻¹, of Encik Hanif's car for the journey
       (i) from Kota Bharu to Kuala Terengganu.
       (ii) from Kuala Terengganu to Setiu.
       (iii) from Setiu to Kota Bharu.
       State the answers correct to three significant figures.
   (b) Encik Hanif is a safety-conscious driver who abides by the speed limit. Is this statement true? State your reasons.

Self Mastery

1. The picture shows three planets in the Solar System.

Mercury	Neptune	Jupiter
[Diameter = 4 879 km]	[Diameter = 49 244 km]	[Diameter = 139 822 km]
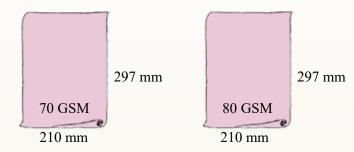
   (a) Calculate the surface area, in km², of all three planets. State the answers in standard form correct to three significant figures.
       [Surface area of sphere = $4\pi r^2$ and $\pi = 3.142$]
   (b) Based on your answer in (a), calculate the difference in surface area between the largest and smallest planets in the Solar System. State the answer correct to four significant figures.

2.

297 mm

70 GSM

210 mm

297 mm

80 GSM

210 mm

The diagram above shows two types of A4-sized paper with different masses. GSM means grams per square metre.
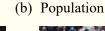
Calculate the mass of one piece of A4-sized paper, in grams for

(a) 70 GSM   (b) 80 GSM

State the answers in standard form correct to three significant figures.

P R O J E C T

1. Look at the pictures below. Obtain the data relevant to the required measurement. Your answers should be in standard form.

2. You can surf various websites or refer to reference books to obtain interesting data related to the pictures below.

    (a) Mass   (b) Population
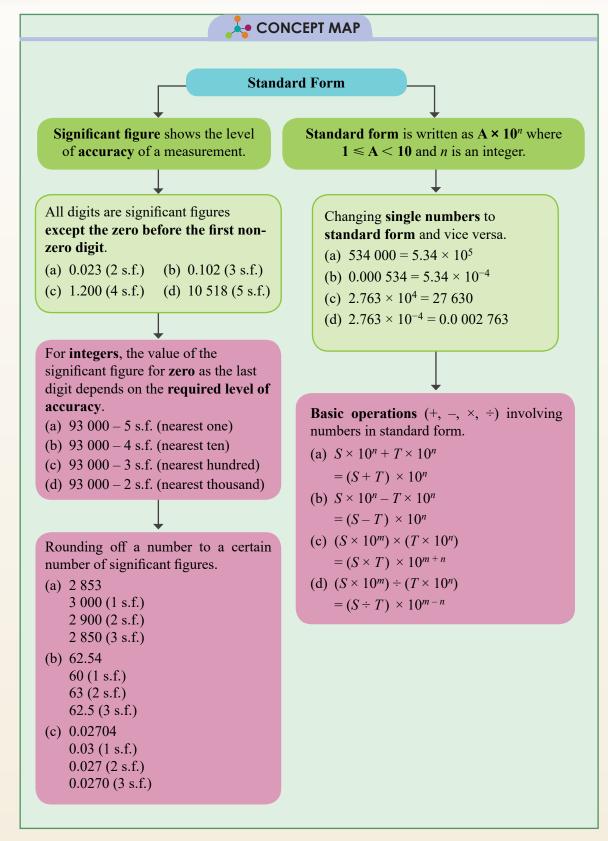
    (c) Distance   (d) Magnitude

3. Obtain other interesting facts that involve calculations in standard form.

4. Present your findings using multimedia applications.

CHAPTER 2

CONCEPT MAP

Standard Form

Significant figure shows the level of **accuracy** of a measurement.

Standard form is written as $A \times 10^n$ where $1 \leqslant A < 10$ and n is an integer.

All digits are significant figures **except the zero before the first non-zero digit**.
(a) 0.023 (2 s.f.)   (b) 0.102 (3 s.f.)
(c) 1.200 (4 s.f.)   (d) 10 518 (5 s.f.)

Changing **single numbers** to **standard form** and vice versa.
(a) $534\ 000 = 5.34 \times 10^5$
(b) $0.000\ 534 = 5.34 \times 10^{-4}$
(c) $2.763 \times 10^4 = 27\ 630$
(d) $2.763 \times 10^{-4} = 0.0\ 002\ 763$

For **integers**, the value of the significant figure for **zero** as the last digit depends on the **required level of accuracy**.
(a) 93 000 – 5 s.f. (nearest one)
(b) 93 000 – 4 s.f. (nearest ten)
(c) 93 000 – 3 s.f. (nearest hundred)
(d) 93 000 – 2 s.f. (nearest thousand)

Basic operations $(+, -, \times, \div)$ involving numbers in standard form.
(a) $S \times 10^n + T \times 10^n$
$= (S + T) \times 10^n$
(b) $S \times 10^n - T \times 10^n$
$= (S - T) \times 10^n$
(c) $(S \times 10^m) \times (T \times 10^n)$
$= (S \times T) \times 10^{m+n}$
(d) $(S \times 10^m) \div (T \times 10^n)$
$= (S \div T) \times 10^{m-n}$

Rounding off a number to a certain number of significant figures.
(a) 2 853
3 000 (1 s.f.)
2 900 (2 s.f.)
2 850 (3 s.f.)
(b) 62.54
60 (1 s.f.)
63 (2 s.f.)
62.5 (3 s.f.)
(c) 0.02704
0.03 (1 s.f.)
0.027 (2 s.f.)
0.0270 (3 s.f.)

(SELF-REFLECT)

At the end of this chapter, I can:

		😀	😞
1.	Explain the meaning of significant figure and thus determine the number of significant figures of a number.		
2.	Round off a number to a certain number of significant figures.		
3.	Recognise and write numbers in standard form.		
4.	Perform basic arithmetic operations involving numbers in standard form.		
5.	Solve problems involving numbers in standard form.		
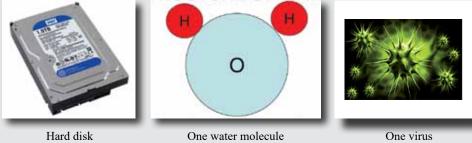
⚙ EXPLORING MATHEMATICS

1. Get into groups.

2. By using the various sources available, identify several measurement values in daily life that are very small or very big. For example,

| Hard disk (1 terabyte) | One water molecule (0.1 nanometer) | One virus (1 micrometer) |

3. Prepare a report on your findings using multimedia applications.

4. Present your report.

5. Obtain additional information from the presentations by other groups.

6. Discuss the advantages of using standard form in various fields.

CHAPTER 3

Consumer Mathematics: Savings and Investments, Credit and Debt

What will you learn?

3.1 Savings and Investments

3.2 Credit and Debt Management

Why do you learn this chapter?

- Knowledge of savings and investments could help us to manage our finances.
- The concept of savings and investments is used in banking, stocks, real estate, business, finance, accounting and so on.

"Many a little makes a mickle".
The above proverb means it is a good habit to save money for the future. Saving habits that have been practised since childhood can help a person cope with any emergency. Investments made by an individual must be in a timely manner in accordance with the current market.

The barter system was practised before the use of money in the economy and was the earliest form of business in the world.

History of money development began with the evolution of the human civilisation itself, which was about 2 000 BC.

http://bukutekskssm.my/Mathematics/F3/
ExploringEraChapter3.pdf

WORD BANK

- liquidity
- interest
- debt
- interest rate
- credit
- investment
- loan
- personal loan
- return
- saving

- *kecairan*
- *faedah*
- *hutang*
- *kadar faedah*
- *kredit*
- *pelaburan*
- *pinjaman*
- *pinjaman peribadi*
- *pulangan*
- *simpanan*

3.1 Savings and Investments

✎ What are savings and investments?

Savings refer to excess money deposited in the safe, money box or drawer. Extra money can also be deposited at a bank that will provide returns based on interest rates and savings periods. There are some common ways of saving in the bank.

Savings Account

- The savings account holder can save any amount according to his ability.
- The account holder receives the interest rate based on the total amount of savings and savings period.
- Interest rates are lower compared to fixed deposit accounts.
- The account holder can withdraw the savings at any time.
- The savings can be withdrawn by using a debit card via an automatic teller machine (ATM).

BULLETIN 📢

Credit counselling and credit agencies encourage each individual to save 10% of their monthly income.

Fixed Deposit Account

- Save a certain amount of money for a certain period of time such as 3 months, 9 months or 1 year tenure.
- Account holders will be offered more competitive interest rates compared to savings accounts.
- Savings cannot be withdrawn before the maturity date.
- If the money is withdrawn before maturity, then the actual interest rate that should be received will be reduced and will be cancelled at a certain time.
- A savings certificate will be issued to the account holder.

Current Account

- Savings in current account can be used for personal or business purposes.
- The account holder may make payment to another party by cheque.
- Savings in the account will not be paid interest and is subjected to service charges. However, there are banks that pay interest to current account holders subject to the bank.
- The current account applicant must submit a referral who is an existing current account holder at the same bank to open the account.
- In addition to cheques, normal withdrawals are usually allowed via debit cards and other channels such as Internet banking, telephone banking and so on.
- The account holder can enjoy the overdraft facility, that is withdrawing money beyond the balance of the deposit, but with interest charges.

Investment is an alternative step for future returns in the form of current income and capital gains. Types of investments are as follows:

Shares

A company will issue shares for the purpose of raising capital. An individual who purchases shares from a company is the owner of the company under certain conditions. The shareholders will receive returns in the form of dividends and capital gains.

Unit Trust

Trust fund is controlled by a unit trust company that is managed by a qualified professional manager in the field of investment. Those who have no knowledge of the purchase of shares can get help from the unit trust companies to manage their money. Unit trust companies collect money from investors and the money is invested in various potential companies with the aim of providing returns that benefit investors.

Real Estate

Investments on immovable assets such as residential houses, shops, land and others are investments in real estate. Investors should consider various aspects before investing.

Factors to be considered in real estate investment are economic situations, income-generating capabilities that is rent, location and property prospects in the future. Individuals who invest in real estate will receive an investment return in the form of rent and capital gains.

BULLETIN

There are two types of investors, that is aggressive and moderate investors:

♦ **Aggressive investors** – invest in stock market.
♦ **Moderate investors** – buy unit trusts, bonds and equity funds.

TIPS

Luxury Stocks (Blue Chip Stocks) are stocks of large companies with track records of excellent business such as Maybank, TNB and Petronas.

BULLETIN

♦ Investment returns comprise current income and capital gains.
♦ Return on current income – rents, dividends, bonus shares.
♦ Capital gains – additional or increased investment value from its original amount. For example, stock prices rose from RM2.00 to RM2.20, thus the addition of RM0.20 was an increase in investment value.

QUIZ

Is the purchase of life insurance and health insurance considered as an investment or savings?

CHAPTER 1

Brainstorming 1

In groups

Aim: To identify types of savings and investments.

Steps:

1. Get into groups of five or six students. Each group should state the types of savings and investments according to the statements given and explain the characteristics of the savings and investments specified.

2. The information collected should be presented in the form of reports as shown below.

	Types of savings	Types of investments	Description
1. Encik Rizal saves a total of RM300 in the bank.	Savings		Savings account – the amount of money saved is small and will be withdrawn at any time.
2. Cik Zeti is a dealer who keeps a sum of money in the bank with the intention of issuing a cheque to pay the creditor.			
3. Mrs Rani uses the money received from her father to buy a shoplot.			
4. Puan Faridah saved a sum of RM20 000 in the bank to finance the education of her children in the future.			
5. Mr Lee bought 1 000 units of unit trusts.			
6. Ms Sharon bought 4 000 units of Bank Orkid Berhad shares worth RM1.00 per share on the Kuala Lumpur Stock Exchange.			

Discussion:

State the advantages and disadvantages of each type of savings and investments stated.

From Brainstorming 1, it is found that savings and investments are different.

MIND TEST 3.1a

1. What is the purpose of an individual's savings?

2. Your father has RM5 000 and has not used it for a long time. What is your advice to him? Explain your answer.

3. Besides merchants, why aren't most people interested in opening current account?

✏️ What do you understand about the benefits of savings?

Interest for savings are rewards paid by financial institutions such as banks to depositors. The interests can be divided into two types, namely **simple interest** and **compound interest**.

🔢 Simple interest

Simple interest is a reward given to the depositor at a certain rate on the deposit amount (principal) for a certain period of time (in years).

Example 1

Encik Zainal deposited RM4 000 at Bank Bunga Raya with interest rate of 2% per annum. How much is the interest earned by Encik Zainal after 1 year?

Solution:

The principal deposited by Encik Zainal is RM4 000. So, 2% of RM4 000 is
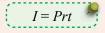
$$\text{RM4 000} \times \frac{2}{100} = \text{RM80}$$

After 1 year, the interest earned by Encik Zainal is

Interest = RM80 × 1
         = RM80

TIPS 💡

Rate is given in percentage form. Thus, we must divide the rate by a hundred. If the time is given in months, do not forget to change to year by dividing by 12 months.

The simple interest can be calculated using the following formula:

$$I = Prt$$

I is the interest, P is the principal, r is the rate and t is the time in years.

Example 2

Encik Badrul deposits RM5 000 in a bank with interest rate of 3% per annum for a period of 2 years. Calculate the total interest that Encik Badrul will receive for the 2-year period.

Solution:

$P = 5\ 000 \qquad r = 3\% = \dfrac{3}{100} = 0.03 \qquad t = 2$

Thus, interest $I = Prt$

$$= \text{RM5 000} \times \frac{3}{100} \times 2$$

$$= \text{RM300}$$

Example 3

Ms Wong deposits RM10 000 in Bank Murni with interest rate of 4% per annum. Calculate the amount of interest Ms Wong will have after 6 months?

Solution:

$I = Prt$

$= RM10\ 000 \times \dfrac{4}{100} \times \dfrac{6}{12}$

$= RM200$

What is the impact on the simple interest as a result of changing the savings period?

The changes in savings period in bank give different returns.

Example 4

Encik Nazrin deposits RM8 000 in Bank Desa with interest rate of 3% per annum. Calculate the total savings of Encik Nazrin after he has saved for

(a) 2 years   (b) 3 years

Solution:

The formula for calculating the interest, $I = Prt$

Principal   = RM8 000
Interest rate   = 3%

(a) 2 years	(b) 3 years
Interest $= RM8\ 000 \times \dfrac{3}{100} \times 2 = RM480$	Interest $= RM8\ 000 \times \dfrac{3}{100} \times 3 = RM720$
Total savings at the end of the second year = RM8 000 + RM480 = RM8 480	Total savings at the end of the third year = RM8 000 + RM720 = RM8 720

Based on the above example, it is found that the longer the savings period (at the bank), the higher the amount of interest earned. Therefore, the final amount of savings also increases.

What is the impact if the given interest rates differ for the same principal?

Dad, what is the impact on the total annual returns if interest rates are different for the same principal?

The total annual returns received will surely be different.

Example 5

Mrs Vanmathy deposits a sum of RM5 000 in a bank. What is the amount of Mrs Vanmathy's savings after 1 year if the interest rate given is
(a) 5% per annum   (b) 6% per annum

What is the difference between the amounts of interest earned by Mrs Vanmathy in the above situations?

QUIZ

Why is a fixed deposit account given higher interest rates than a savings account?

Solution:

Total savings	Interest rate	Savings period (years)	Total interest	Total savings after 1 year
RM5 000	5%	1	$RM5\ 000 \times \dfrac{5}{100} \times 1$ $= RM250$	RM5 000 + RM250 $= RM5\ 250$
RM5 000	6%	1	$RM5\ 000 \times \dfrac{6}{100} \times 1$ $= RM300$	RM5 000 + RM300 $= RM5\ 300$

The difference between the total interests received is RM300 − RM250 = RM50.

Based on Example 5, for the same principal, when the interest rates increase, the total savings at the end of the year also increase.

TIPS

↑ interest rate   ↑ the higher the return

↓ interest rate   ↓ the lower the return

Compound interest

Compound interest is interest that is calculated based on the original principal and also the accumulated interest from the previous period of savings.

Compound interest is different from simple interest in terms of the amount of savings to be used for interest calculation.

For compound interest, the frequency of compounding on the principal can be different. For example, compounded once a year or once every 3 months and so on.

Referring to Example 4(a), if Encik Nazrin is given compound interest with compounding once a year, what is his savings at the end of the second year?

In the first year, the amount of interest received is

$$RM8\ 000 \times \frac{3}{100} = RM240.$$

Thus, the amount of savings at the end of the first year is RM8 240.
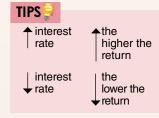
For the second year, the amount of savings used for interest calculation is RM8 240 (principal + first year interest).
Thus, interest at the end of the second year is

$$RM8\ 240 \times \frac{3}{100} = RM247.20.$$

Scan the QR Code or visit http://bukutekskssm. my/Mathematics/F3/ Chapter3Compund.pdf for more information about compound interest.

Therefore, the amount of Encik Nazrin's savings at the end of the second year is

$$RM8\ 240 + RM247.20 = RM8\ 487.20.$$

CHAPTER 1

57
KPM

In general, the formula for calculating compound interest is:

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$

MV = matured value
P = principal
r = yearly interest rate

n = number of periods the interest is compounded per year
t = term in years

Based on Encik Nazrin's example, it was found that;
$$P = 8\ 000, \quad r = 0.03, \quad n = 1, \quad t = 2.$$
Thus, the amount of Encik Nazrin's savings at the end of the second year is

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= RM8\ 000\left(1 + \frac{0.03}{1}\right)^{(1)(2)}$$
$$= RM8\ 000\ (1.0609)$$
$$= RM8\ 487.20$$

Example 6

At the beginning of a year, Mrs Liew Foong saves RM15 000 in savings account with a rate of 4% per annum and compounded every 6 months. What is Mrs Liew Foong's total savings at the end of the third year?

Solution:

$$P = 15\ 000 \qquad r = \frac{4}{100} = 0.04 \qquad n = 2 \qquad t = 3$$

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$
$$= 15\ 000\left(1 + \frac{0.04}{2}\right)^{(2)(3)}$$
$$= 15\ 000\ (1.1262)$$
$$= RM16\ 892.44$$

QUIZ

What is the impact on the total cumulative returns, if the compounding rate increases in a year?

Example 7

A bank offers 5% interest rate per annum for savings in fixed deposit account. If Puan Wahidah saves RM10 000 at the beginning of the year, how much money is in her fixed deposit account at the end of the year if interest is compounded
(a) once every 3 months?   (b) once a month?

Solution:

$$P = 10\ 000 \qquad r = \frac{5}{100} = 0.05 \qquad t = 1$$

$$MV = P\left(1 + \frac{r}{n}\right)^{nt}$$

BULLETIN

Inflation also affects the value of the currency. If the inflation rate increases, the purchasing power of RM1 will reduce.

(a) $n = 4$
Thus,
$$MV = 10\,000 \left(1 + \frac{0.05}{4}\right)^{(4)(1)}$$
$$= RM10\,509.45$$

(b) $n = 12$
Thus,
$$MV = 10\,000 \left(1 + \frac{0.05}{12}\right)^{(12)(1)}$$
$$= RM10\,511.62$$

From Example 7, it was found that when the compounding frequency increases, the future value of savings also increases.

Example 8

Mr Charles deposited RM6 000 in a fixed deposit account at Bank Berjaya for 2 years with interest rate of 6% per annum. What is the difference between the amount of interests Mr Charles earned if he was given compound interest (compounded once every 4 months) compared to simple interest?

Solution:

Simple interest	Compound interest
Interest, $I = Prt$ $= RM6\,000 \times \frac{6}{100} \times 2$ $= RM720$	$MV = P\left(1 + \frac{r}{n}\right)^{nt}$ $= 6\,000\left(1 + \frac{0.06}{3}\right)^{(3)(2)}$ $= RM6\,756.97$ Total accumulated interest RM6 756.97 – RM6 000 = RM756.97
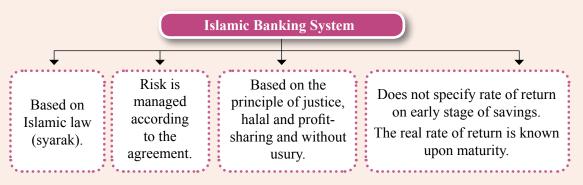
Thus, the difference in the amount between simple interest and compound interest (with the frequency of 4 months) is

RM756.97 – RM720 = RM36.97

Based on Example 8, it is clear that savings with compound interest give higher returns than savings with simple interest.

Islamic Banking

Malaysia practises dual banking systems. They are conventional banking system and Islamic banking system.

Islamic Banking System

- Based on Islamic law (syarak).
- Risk is managed according to the agreement.
- Based on the principle of justice, halal and profit-sharing and without usury.
- Does not specify rate of return on early stage of savings. The real rate of return is known upon maturity.

Example 9

Encik Osman saved RM20 000 in a savings account in an Islamic bank, according to the principle of wadiah for 1 year. By the end of the year, he received a sum of RM20 500 as a return from the savings. An additional RM500 is a hibah (gift) from the bank. Calculate the percentage of hibah obtained by Encik Osman.

Solution:

$$\text{Percentage of hibah} = \frac{\text{RM500}}{\text{RM20 000}} \times 100\% = 2.5\%$$

2.5% is only a reference for savings and is not fixed.

BULLETIN

The principle of wadiah
Property or cash received with the agreement of the customer to be deposited in the bank. The bank is responsible for the security of the property or the money.

MIND TEST 3.1b

1. Puan Nathania deposited RM500 into her savings account that gives an interest rate of 4% per annum and compounded quarterly. How much is Puan Nathania's savings at the end of the fifth year?

2. Mr Chong deposited RM1 000 into his savings account that gives an interest rate of 5% per annum and compounded every half year. How much is Mr Chong's savings at the end of the third year?

3. Puan Aminah deposited RM100 into her savings account that gives an interest rate of 3% per annum and compounded monthly. How much is Puan Aminah's savings at the end of the second year?

✎ What do you understand by return on investment (ROI)?

LEARNING STANDARD

Perform calculations involving the value of return on investments, and hence explain the factors that affect the return on investments and its impacts.

Return on investment refers to the return value of each ringgit invested by the investor. In other words, return on investment is also a ratio of profit or loss derived from an investment.

In general, investors prefer to assess the return on investment in percentage. Return on investment will reflect the profit or loss achieved by individual investors in investment.

An investment is considered profitable (wise investment) when the present value of the investment and the amount of return received is more than the value of the original investment.

Similarly, when the amount of return and the present value of the return is less than the value of the original investment, then the investment is unprofitable.

BULLETIN

School cooperatives declare dividends at the end of each financial year. The declared dividend determines the return value of the shares purchased by each member of the cooperative.

The formula for calculating return on investment is

$$\text{Return on investment} = \frac{\text{Total return}}{\text{The value of the initial investment}} \times 100\%$$

In addition, investors also have an expected rate of return from an investment. For example, an investor expects a rate of return of 10% of their investment. However, the real rate of return to be received may not be as expected.

Investment instruments consist of the unit trust, shares, real estate and so on. Each of these investment instruments will bring returns.
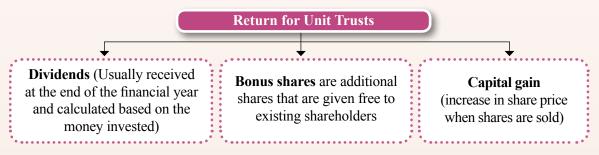
🔢 Unit Trust

Unit trust is a good investment alternative for medium term investment (3 to 5 years) and long term (over 5 years).

Investments in unit trusts are low risk as they are managed by professional fund managers regulated by the securities commission and also monitored by Bank Negara Malaysia.

Investments in unit trusts allow investors to diversify their investments with small capital.

Below is the return for unit trusts.

Return for Unit Trusts

Dividends (Usually received at the end of the financial year and calculated based on the money invested)	**Bonus shares** are additional shares that are given free to existing shareholders	**Capital gain** (increase in share price when shares are sold)

Example 10

On 1 January 2018, Puan Siti invested 3 000 units valued at RM2.00 per unit in Amanah Saham Bumiputera (ASB). For the financial year ending 31 December 2018, Amanah Saham Bumiputera paid a dividend of 5%. On 1 January 2019, Puan Siti sold all the shares she owned at RM2.20 per unit. What is the return on investment for Puan Siti?

Solution:

Steps for calculating dividend
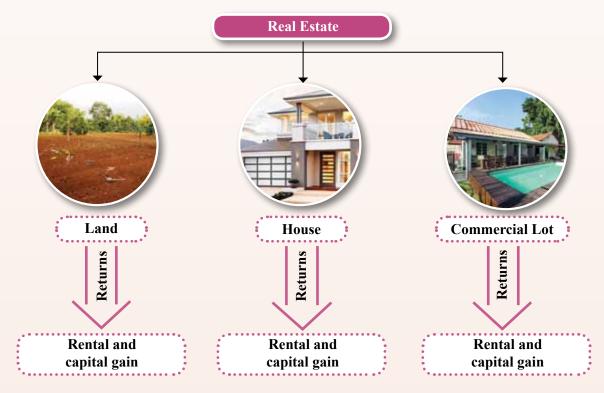
Initial capital $= 3\ 000 \times$ RM2.00
$\qquad\qquad = $ RM6 000

Dividend $= \dfrac{5}{100} \times (3\ 000 \text{ units} \times \text{RM2.00})$
$\qquad\quad = $ RM300

Increase in share price = RM2.20 – RM2.00 = RM0.20
Capital gain $=$ RM0.20 $\times$ 3 000 units
$\qquad\qquad = $ RM600
Total return $=$ RM300 + RM600 = RM900

Return on investment $= \dfrac{\text{RM900}}{\text{RM6 000}} \times 100\% = 15\%$

The return on investment benefits Puan Siti as she receives two types of returns namely dividend and capital gain from an increase in share values from RM2.00 to RM2.20.

📊 What do you understand about return on investment for real estate?

Investment in real estate is one of the investments that bring returns in the form of rent and capital gains. When a property is rented, the owner (investor) of the property will receive return in the form of rent. If the property is sold, the owner (investor) will receive capital gain or capital loss.

Real Estate

Land	House	Commercial Lot
Returns	Returns	Returns
Rental and capital gain	Rental and capital gain	Rental and capital gain

Example 11

Encik Yusuf bought a shoplot at a price of RM600 000 on 1 January 2017 in Bangi. He paid 10% of the shoplot's purchase price of RM60 000. The shoplot was rented from 1 January 2017. On 31 December 2026, he sold the shoplot for RM1 300 000. The loan amount still owed to the bank was RM486 000. Meanwhile, the amount that has been amortized to the bank was RM450 000. Other charges involved in the sale and purchase transactions are as follows:

Legal cost	RM15 000
Stamp duty (during sale and purchase)	RM15 000
Agent's commission	RM18 000

BULLETIN 📢

Stamp duty
Tax imposed on documents or letters with legal, commercial or financial implications under the First Schedule, of Stamp Act 1949.

Legal cost
Payment to lawyer to perform the transfer of property for the buyer.

Commission
Fee paid by the property seller to the agent for the sale of real estate.

The total rent collected throughout the possession of the shoplot is RM200 000. Calculate the return on investment obtained by Encik Yusuf.

Solution:

Total rent = RM200 000

Capital gain = RM1 300 000 – RM486 000 – RM60 000 – RM15 000 – RM15 000
– RM18 000 – RM450 000
= RM256 000

Total return = RM200 000 + RM256 000
= RM456 000

Return on investment = $\dfrac{\text{RM456 000}}{\text{RM600 000}} \times 100\%$

= 76%

Example 12

Encik Hussein bought a house on 1 January 2015 in Cheras at RM300 000 and cleared 10% down payment of RM30 000. He expects a return of 30% over 20 years.

Encik Hussein sold the house at a price of RM600 000 after having owned the house for 20 years. The loan amount amortized to the bank was RM475 000. During that period, he managed to earn a rent of RM60 000. Other expenses incurred are as follows:

Stamp duty (during sale and purchase)	RM4 000
Agent's commission	RM2 000
Legal cost during sale and purchase	RM4 000

What is the return on investment for Encik Hussein for 20 years? Did he achieve his desire to get a return of 30%?

Solution:

Return on investment
= Rent + Capital gain
= RM60 000 + (RM600 000 – RM30 000 – RM475 000 – RM4 000 – RM2 000 – RM4 000)
= RM60 000 + RM85 000
= RM145 000

Return on investment = $\dfrac{\text{RM145 000}}{\text{RM300 000}} \times 100\%$

= 48.33%

Encik Hussein managed to obtain a rate of return of 48.33%. This rate exceeds the expected rate of return of 30%.

In real estate investment there are factors that affect the return on investment. The factors that affect return on investment are as follows:

The economic situation
- The country's good economic situation will increase real estate prices because the demand for real estate will increase.

Location
- The properties that are strategically located near a vastly developed city centre have higher prices compared to properties in rural areas.

Factors that affect the return on investment on real estate

Political situation
- A stable political situation will increase demand for real estate.
- This will indirectly increase real estate prices.
- Political instability will reduce demand for real estate and indirectly cause the fall of real estate prices.

MIND TEST 3.1c

1. What is the meaning of return on investment or ROI?

2.

- On 1 January 2019, Mr Moses bought a homestay at a price of RM250 000.
- The daily rental rate is RM100.
- On average, the homestay will be occupied for 20 days in a month.
  (a) Calculate the monthly rentals.
  (b) Calculate the return on investment if the homestay is sold for RM480 000 by the end of the year.

3. On 1 January 2018, Rahim invested 4 000 units valued at RM1 per unit in Amanah Saham Bumiputera (ASB). For the financial year ending 31 December 2018, ASB paid dividend of 8%.

   How much is the dividend received by Rahim for that year?

✏ **What factors should be considered before making an investment?**

> **LEARNING STANDARD**
>
> Compare and contrast potential risks, return and liquidity of various types of savings and investments.

Three factors that should be taken into account by an investor prior to making an investment are as follows:

Potential investment risks	The possibility of an uncertainty that may incur from the investments made.
The level of returns	Profit enjoyed by investors from investments.
Liquidity aspects	Relating to how soon the investment or savings could be cashed out.

CHAPTER 1

The table below shows the comparison of various types of savings and investments by individuals in terms of risk levels, return levels and liquidity levels.

Types of investments	Risk levels	Return levels	Liquidity levels
Saving	Risk free	Low	High
Fixed deposits	Risk free	Low	High
Company shares	High	High	Moderate
Real estate	Moderate	High	Low
Unit trust	Low	Moderate	High

MIND TEST 3.1d

1. Explain the relationship between risk and return on investment.

2. The potential risk of saving in a bank is zero as compared to investment. Explain this statement.

3. Why do unit trusts have a high liquidity level?

4. Real estate has a moderate potential risk. Explain.

5.

• **RM150 per day**

Encik Osman sets up a homestay on the lot of land purchased at RM250 000. The overall cost of setting up this homestay is RM500 000.

(a) What type of investment was made by Encik Osman?

(b) State the potential risks, return and liquidity of the investment made by Encik Osman.

(c) In your opinion, was the investment made by Encik Osman a wise move? Justify.

✎ What do you understand about cost averaging strategy?

Cost averaging strategy is a technique commonly practised by investors who invest in shares with fixed amount for a certain period such as monthly, quarterly or yearly regardless of the stock market conditions.

Cost averaging strategies can help investors to buy shares with lower average cost and the total number of shares owned will be higher within the same investment period, as opposed to buying them in a lump sum or with a single purchase.

For example, Puan Hasniza has RM12 000 and buys shares in a lump sum from Sempurna Company at RM2.00 per share unit. Hence, Puan Hasniza will only have 6 000 units of shares (6 000 units of shares × RM2.00 = RM12 000). However the situation will be different if Puan Hasniza decides to buy shares based on cost averaging strategy.

The table below shows Puan Hasniza's investment chart in a year based on cost averaging strategy for buying shares on a monthly basis.

Month	Investment amount (RM)	Unit price (RM)	Number of units $=\dfrac{\text{Investment amount}}{\text{Unit price}}$
January	1 000	2.00	500
February	1 000	1.80	555
March	1 000	1.80	555
April	1 000	1.70	588
May	1 000	1.70	588
June	1 000	1.60	625
July	1 000	1.60	625
August	1 000	1.50	666
September	1 000	1.60	625
October	1 000	2.20	454
November	1 000	2.30	434
December	1 000	1.90	526
	12 000	1.78 (average cost per share unit)	6 741 (number of units owned)

With the cost averaging strategy given above, Puan Hasniza earned 6 741 units by investing RM12 000.

Cost averaging strategy

$$\text{Average Cost} = \dfrac{\text{Investment amount}}{\text{Number of share units owned}}$$

$$\text{Average cost per share unit} = \dfrac{\text{RM12 000}}{6\ 741} = \text{RM1.78}$$

TIPS 💡

Cost averaging strategy allows a person not to buy shares at a high price or buy shares before it reaches the lowest price.

Example 13

Below are two investors who plan to invest in shares using different strategies.

I would like to invest RM20 000 to buy shares and would accumulate the shares in the designated months by spending the same amount of money.

I want to invest a lump sum of RM20 000 to buy Pelita Company shares at RM2.00 a unit.

Mrs Esther Wong

Puan Linda

Shares acquired by Mrs Esther Wong in the designated months:

Month	January	March	May	August	December
Shares price per unit (RM)	2.00	1.80	1.60	2.10	2.00

(a) Calculate the average cost per unit and the number of shares owned by Puan Linda and Mrs Esther Wong.

(b) Who is a wise investor? Justify your answer.

Solution:

(a) Puan Linda

$$\text{Total shares} = \frac{\text{RM20 000}}{\text{RM2.00}} = 10\ 000 \text{ share units}$$

$$\text{Average cost per share} = \frac{\text{RM20 000}}{10\ 000 \text{ share units}}$$
$$= \text{RM2.00}$$

Mrs Esther Wong

Month	Total investment	Price per unit (RM)	Number of share units
January	4 000	2.00	2 000 units
March	4 000	1.80	2 222 units
May	4 000	1.60	2 500 units
August	4 000	2.10	1 904 units
December	4 000	2.00	2 000 units
	20 000		10 626 share units

Total shares = 10 626 share units
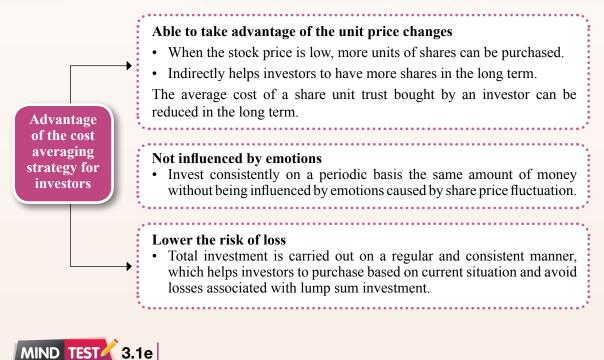
$$\text{Average cost per share} = \frac{\text{RM20 000}}{10\ 626 \text{ units}}$$
$$= \text{RM1.88}$$
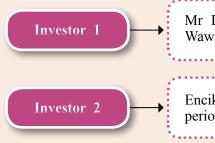
(b) Mrs Esther Wong is a wise investor for practising the cost averaging strategy that helped her to accumulate more shares with the same amount of money.

Able to take advantage of the unit price changes
- When the stock price is low, more units of shares can be purchased.
- Indirectly helps investors to have more shares in the long term.

The average cost of a share unit trust bought by an investor can be reduced in the long term.

Advantage of the cost averaging strategy for investors

Not influenced by emotions
- Invest consistently on a periodic basis the same amount of money without being influenced by emotions caused by share price fluctuation.

Lower the risk of loss
- Total investment is carried out on a regular and consistent manner, which helps investors to purchase based on current situation and avoid losses associated with lump sum investment.

MIND TEST 3.1e

1. What do you understand about the cost averaging strategy in purchasing shares?

2. Below are two investors who invested using different strategies.

Investor 1 → Mr Derick invested a lump sum of RM24 000 to purchase Wawasan shares at RM2.00 per share unit.

Investor 2 → Encik Sulaiman has RM24 000 and invested consistently on a periodic basis RM2 000 each month to purchase Wawasan shares.

Month	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Share price per unit (RM)	2.00	1.80	1.70	1.60	2.10	1.50	2.20	2.00	2.00	1.60	1.70	1.80

The table above shows the share price bought by Encik Sulaiman on a monthly basis.
(a) Who is a wise investor? Justify your answer.
(b) Calculate the average cost per share unit and the number of shares owned by Encik Sulaiman.
(c) State the advantages of using cost averaging strategy in purchasing shares.
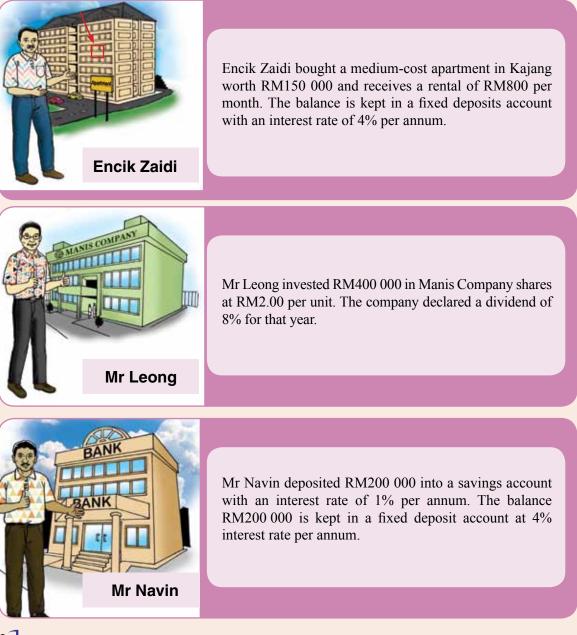
How do you solve problems involving savings and investments?

Example 14

Encik Zaidi, Mr Leong and Mr Navin had retired from their jobs and each received RM400 000 as gratuity from their company. They use different investment methods to invest.

Who is a wise investor?

Encik Zaidi

Encik Zaidi bought a medium-cost apartment in Kajang worth RM150 000 and receives a rental of RM800 per month. The balance is kept in a fixed deposits account with an interest rate of 4% per annum.

Mr Leong

Mr Leong invested RM400 000 in Manis Company shares at RM2.00 per unit. The company declared a dividend of 8% for that year.

Mr Navin

Mr Navin deposited RM200 000 into a savings account with an interest rate of 1% per annum. The balance RM200 000 is kept in a fixed deposit account at 4% interest rate per annum.

Solution:

Encik Zaidi

Level of risk	The level of risk for property (apartments) and fixed deposits is low.		
Return on investment	Receives a return on investment in the form of rent and interest.		
	Rental	**Savings interest**	**Return on investment (ROI)**
	RM800 × 12 = RM9 600	$\frac{4}{100}$ × RM250 000 = RM10 000	**ROI** = $\frac{RM19\ 600}{RM400\ 000}$ × 100% = 4.9%
Liquidity	Savings are easily converted to cash while property takes time to sell.		

Mr Leong

Level of risk	The level of risk in buying shares is high.
Return on investment	Receives **dividends** and bonuses depending on the **performance** of the company invested in. If he sells shares, he is uncertain that he will be able to sell the shares at a higher price compared to the purchase price. This depends on the economic situation and performance of the company stock at that period. Dividend = 400 000 × $\frac{8}{100}$ **ROI** = $\frac{RM32\ 000}{RM400\ 000}$ × 100% = RM32 000 = 8%
Liquidity	Moderate.

Mr Navin

Level of risk	The level of risk for both savings and fixed account are low.		
Return on investment	Receives a return on investment in the form of interest only.		
	Savings interest	**Fixed deposit interest**	**Return on investment (ROI)**
	RM200 000 × $\frac{1}{100}$ = RM2 000	$\frac{4}{100}$ × RM200 000 = RM8 000	**ROI** = $\frac{RM10\ 000}{RM400\ 000}$ × 100% = 2.5%
Liquidity	Savings are easily turned into cash.		

- Mr Leong is a wise investor because his returns on investment value is higher than Encik Zaidi and Mr Navin.

- In this example, Encik Zaidi, Mr Leong and Mr Navin each uses the same amount of capital, which is RM400 000. The effectiveness of their investments can be compared based on their returns for the year respectively.

CHAPTER 1

1. The following are two investors who invested their gratuity.

Mr Rasamanie received RM600 000 as gratuity from his company. He bought a double storey shop in Bangi, Selangor and receives a monthly rental of RM3 500.

Monthly rental
RM3 500

Encik Nik Izwan received RM600 000 as gratuity from his company. He saves RM150 000 in a fixed deposit account at a commercial bank with an interest rate of 4% per annum. He also bought share units worth RM150 000.

At the same time, Encik Nik Izwan bought shares in Cepat Maju Company worth RM100 000. The balance of the gratuity is used to buy a medium-cost apartment in Ampang and receives a monthly rental of RM1 200.

Monthly rental
RM1 200

(a) Explain the investment risk level of both individuals.
(b) Who is a wise investor? Justify your answer.
(c) What factors need to be considered before investing in real estate?

2. In 2015, Mr Wong bought a house at RM540 000. He paid 10% down payment and the balance is paid through a loan. After 20 years, Mr Wong decided to sell the house at RM900 000. The following are the expenses involved.

Monthly instalment amount paid	RM666 000
Stamp duty	RM15 000
Agent's commission	RM8 000
Other expenses	RM18 000

Calculate the return on investment for Mr Wong.

3.2 Credit and Debt Management

📝 What do you understand about credit and debt?

The word credit has several meanings. In the financial world, **credit** means a contractual agreement between the supplier (for instance bank or financial institution) and consumer. The consumer can borrow money from the supplier for any use or purchase and agree to repay within a certain period. In short, credit is a postponement of payment facility provided by the supplier to the consumer.

For example, banks offer credit facilities to customers in the form of credit cards. If the customer or credit card owner uses the credit card in a transaction, the bank will pay the seller first and the customer will pay back the bank within a certain period of time.

Credit can also mean the amount of money that can be borrowed. For example, for credit card, if the credit card limit is RM10 000, then the card owner has the ability to buy goods or make transactions up to RM10 000 with the card.

Debt usually means an amount that has been borrowed but has not been settled. If a transaction is made using a credit card, the credit will be converted into debt.

Personal budget is the estimated income and expenditure of an individual for a given period. The practice of making personal budget is strongly encouraged so that an individual can

(a) plan spending prudently

(b) avoid overspending

(c) save

🧮 How do you manage credit and debt wisely?

- Credit card users need to settle the debt payment within the period stipulated by the bank to enjoy interest free period.
- Pay the outstanding balance listed on the credit card statement.
- The minimum amount paid by the credit card holder provides opportunity for the bank to charge interest on the balance and may also incur late payment charges.
- Pay within the cash discount period for payment of debts.

Example 15

Encik Syed bought an air conditioner at RM3 200 on 15 July 2018. He lacked RM1 200 in cash but has a Bank Cemerlang credit card. He was aware that the shortage of cash could be paid at the end of the month when he received his salary.

(a) Which credit facility can be used by Encik Syed to overcome the shortage of money?

(b) State the advantages and disadvantages of the payment method you specified in answer (a).

Solution:

(a) Encik Syed can use the credit card facility.

(b) The use of credit cards is more convenient if Encik Syed repays his credit within the interest-free period to avoid any extra charges.

Brainstorming 2
In groups

Aim: The impact of using credit card in buying goods online.

Steps:

1. Get into groups of four or five.
2. Surf any website related to the topics of discussion for more information.
3. The information collected should be presented using an appropriate thinking map to the class.
4. The best thinking map will be displayed in the mathematics corner.

Discussion:
What is the impact of purchasing goods online?

From Brainstorming 2, it is found that purchasing goods online can cause consumers to spend extravagantly and get into debt. Thus, be prudent when buying goods online.

MIND TEST ✏️ **3.2a**

1. What does a personal loan mean?

2. | Many people are bankrupt due to credit cards |

   What are the ways to overcome the situation above?

3. Puan Zuraidah wants to buy a refrigerator at Hebat Electrical Shop but she lacks of RM2 500 in cash. Hebat Electrical Shop provides instant loans for purchases with an interest rate of 4% per annum. Puan Zuraidah also has a credit card. Which credit facility should be used by Puan Zuraidah and state its advantages?

NICOL REFRIGERATOR
RM3 200
HEBAT ELECTRICAL SHOP

🖊 **What do you understand about the advantages and disadvantages of credit cards?**

LEARNING STANDARD

Investigate and describe the advantages and disadvantages of credit card and ways to use it wisely.

The use of credit cards is increasingly common today. As a consumer it is important for us to realize and understand the advantages and disadvantages of using credit cards.

Advantages of credit cards	Disadvantages of credit cards
• Credit card users can enjoy a reward system in the form of cash rebates or points redemption. • Does not require us to carry a lot of cash. • Easy and efficient payment method. • Convenience of buying goods and services online.	• Incur charges such as annual fees, finance charges (interest), cash advance interest charges and late payment charges. • Overspending. • Some stores do not accept credit payment.

However, not all individuals are eligible for a credit card. There are several conditions that an applicant must adhere to.

- 21 years old and above.
- Minimum income of RM24 000 per annum and meets other requirements set by the bank.
- Requires salary slip or supporting documents.

Credit card users must comply with the obligations as a credit card user when signing the credit card application form.

- Do not give credit card details to strangers.
- Remember the pin number and do not record the pin number on the back of the card.
- Check the transactions in the credit card statement received at the end of the month.

✎ What do you understand about the impact of minimum payment and late payment on credit card use?

The credit cardholder will receive a financial statement for the credit card monthly. In the statement, there are details such as credit limit, statement date, latest amount, minimum payment amount, type of charges and so on.

The cardholder should pay the statement balance immediately so that no financial charges are incurred. But bank provides flexibility by allowing users to pay in a given period, known as the interest free period. Usually this period is 20 days from the statement date.

To enjoy this privilege every month, the cardholder must pay the total balance of credit card statement or make a minimum payment in the interest free period. The minimum payment is usually 5% of the total balance of the credit card statement, or a minimum of RM50.

If there is still a balance of the latest amount upon expiry without interest, the finance charge (or interest) will be imposed on the balance in daily rate. Most banks charge an annual interest rate of between 15% and 18%.

In addition, if no payment is made within the interest free period, then the minimum late payment charge of RM10 or 1% of total outstanding balance as at statement date will be charged.

TIPS

The credit cardholder must pay the full statement balance of the credit card to enjoy a 20-day interest free period from the statement date.

However, if you pay only part of it, you will lose the interest-free period.

Example 16

Encik Ahmad received his credit card statement for January 2019 from Bank Sentosa. The statement shows Encik Ahmad has the current amount (outstanding balance) of RM5 200. Assuming Encik Ahmad did not use his credit card in February.

(a) What is the minimum payment to be paid?

(b) If he only makes a minimum payment for January and the statement date is 15 days from the expiry date of the interest free period, what is the balance for the February statement?

(c) If he missed his payment for January, what is the balance for his February statement?

Solution:

(a) Current amount $= RM5\ 200$

5% of the current amount $= \left(\dfrac{5}{100}\right) \times RM5\ 200 = RM260$

TIPS

We should use credit cards wisely.

This amount exceeds RM50, thus the minimum payment to be paid by Encik Ahmad is RM260.

(b) Outstanding balance $= RM5\ 200 - RM260 = RM4\ 940$
Period subject to financial charges $= 15$ days $= (15 \div 365)$ years
Interest charged $= RM4\ 940 \times [(18 \div 100) \times (15 \div 365)] = RM36.54$

Current amount (Outstanding balance) in February $= RM4\ 940 + RM36.54 = RM4\ 976.54$

(c) Outstanding balance = RM5 200

Period subject to financial charges = 15 days = (15 ÷ 365) years

Interest charged = RM5 200 × [(18 ÷ 100) × (15 ÷ 365)] = RM38.47

Late payment charges $= \left(\dfrac{1}{100}\right) \times$ (RM5 200 + RM38.47) = RM52.38

Current amount in February = RM5 200 + RM38.47 + RM52.38 = RM5 290.85

✎ How do you solve problems involving the use of credit cards?

Credit card users should be aware of the advantages and disadvantages of credit cards. Several factors have to be taken into account before using credit cards such as the balance of the existing limit, current cash flow and so on.

CHAPTER 1

Example 17

Ms Chin wants to buy a French made handbag online. She surfs the Internet and finds two interesting promotions:

(a) Company L in Singapore offers promotional price of SGD250. For orders outside Singapore, SGD50 shipping charges apply.

(b) Company V in Malaysia offers promotional price of RM799. Delivery is free for all orders to local address.

Ms Chin intends to make payment by credit card and she understands that the bank will charge an additional 1% on each transaction from abroad. Assume the current exchange rate for Malaysian ringgit is

> **RM1 = SGD0.34**

As a wise consumer, which offer should Ms Chin choose? Justify your choice.

Solution:

Ms Chin should compare the actual price to be paid if buying from the two companies.

(a) Company L:

Promotional price = SGD250 × (1 ÷ 0.34) = RM735.29

Postal charges = SGD50 × (1 ÷ 0.34) = RM147.06

Additional charges by bank = RM735.29 × $\left(\dfrac{1}{100}\right)$ = RM7.35

Actual price to be paid = RM735.29 + RM147.06 + RM7.35 = RM889.70

(b) Company V:

Promotional price = RM799

Actual price to be paid = RM799

Although the promotional price offered by Company L is cheaper, but the actual price payable is higher due to the additional charges incurred for online purchases from Company L. So, Ms Chin should buy from Company V to save RM90.70.

How do you calculate loan repayment and instalment?

Each loan will be charged interest on the loan from the date the loan was made. There are two types of calculation methods for loan interest, namely flat interest rate and interest on balance.

Total loan balance is the amount deducted from the initial down payment plus the amount of interest charged.

The monthly instalment is the total amount paid by the borrower to the bank every month to settle the balance of the loan.

Flat interest

In the flat interest method, the interest rate will be calculated on the original loan amount over the term of the loan. So the amount of interest charged per month is fixed.

Example 18

Mrs Lim bought a car worth RM80 000 on credit. She pays 10% down payment and the balance is payable in instalments over 6 years. The flat interest rate imposed by the bank is 4% per annum. What is the amount of repayment and monthly instalment payable by Mrs Lim?

Solution:

$$\text{Loan amount} = \text{Purchase price} - \text{down payment}$$
$$= \text{RM80 000} - \text{RM8 000} = \text{RM72 000}$$

$$\text{Interest for 6 years} = \text{RM72 000} \times \frac{4}{100} \times 6 \text{ years} = \text{RM17 280}$$

$$\text{Total repayment} = \text{RM72 000} + \text{RM17 280} = \text{RM89 280} \longleftarrow$$

Total repayment can be calculated with the formula $A = P + Prt$

$$\text{Monthly instalment} = \frac{\text{RM89 280}}{72 \text{ months}} = \text{RM1 240 per month}$$

Example 19

Encik Azlan obtained a personal loan of RM10 000 from Bank Mulia with an interest rate of 4% per annum. The repayment period is 7 years.

What is the monthly instalment to be paid by Encik Azlan?

Solution:

$A = P + Prt$
Loan, $P = $ RM10 000
$\quad\quad r = 4\%$
$\quad\quad t = 7$ years

Thus, total repayment

$$A = \text{RM10 000} + \left(\text{RM10 000} \times \frac{4}{100} \times 7\right)$$
$$= \text{RM10 000} + \text{RM2 800}$$
$$= \text{RM12 800}$$

$$\text{Monthly instalment} = \frac{\text{RM12 800}}{84 \text{ months}} = \text{RM152.38}$$

🔢 Interest on balance

In addition to the flat interest, banks also offer interest on balance for certain types of loans. In the interest method over balance, the amount of interest charged each month on the loan depends on the amount of the loan balance for that month. Since there is monthly instalment payment, the amount of the loan balance will be reduced, thus the amount of interest for each month will also be reduced.

However, it should be noted that for every instalment paid each month, the priority is given to settle the interest amount in that month, and then the outstanding balance is used to settle the balance of the loan amount.

Example 20

Encik Harith obtained a personal loan of RM10 000 from Bank Mulia with an interest rate of 6% on the balance. The repayment period is 7 years while the monthly instalment is RM150.

Calculate the amount of interest payable by Encik Harith for the first three months.

Solution:

First month

$$\text{First month interest} = \text{RM10 000} \times \frac{6}{100} \times \frac{1}{12}$$
$$= \text{RM50.00}$$

$$\begin{aligned}\text{Loan at the end of first month} &= \text{RM10 000} + \text{RM50} \\ &= \text{RM10 050}\end{aligned}$$
$$\begin{aligned}\text{Balance after first instalment} &= \text{RM10 050} - \text{RM150} \\ &= \text{RM9 900}\end{aligned}$$

Second month

Balance of the loan at the beginning of second month = RM9 900

$$\text{Second month interest} = \text{RM9 900} \times \frac{6}{100} \times \frac{1}{12}$$
$$= \text{RM49.50}$$

$$\begin{aligned}\text{Loan at the end of second month} &= \text{RM9 900} + \text{RM49.50} \\ &= \text{RM9 949.50}\end{aligned}$$
$$\begin{aligned}\text{Balance after second instalment} &= \text{RM9 949.50} - \text{RM150} \\ &= \text{RM9 799.50}\end{aligned}$$

Third month

Balance of the loan at the beginning of third month = RM9 799.50

$$\text{Third month interest} = \text{RM9 799.50} \times \frac{6}{100} \times \frac{1}{12}$$
$$= \text{RM49.00}$$

$$\begin{aligned}\text{Loan at the end of third month} &= \text{RM9 799.50} + \text{RM49.00} \\ &= \text{RM9 848.50}\end{aligned}$$
$$\begin{aligned}\text{Balance after third instalment} &= \text{RM9 848.50} - \text{RM150} \\ &= \text{RM9 698.50}\end{aligned}$$

Total interest for the first three months is RM50.00 + RM49.50 + RM49.00 = RM148.50

Scan the QR Code or visit http://bukutekskssm. my/Mathematics/F3/ Chapter3InterestRate Calculation.pdf for more information about loan with flat interest and interest on debts.

✎ How do you solve problems involving loans?

Example 21

Ameera wants to buy a car and has paid a deposit of RM4 800. The balance will be settled through a vehicle loan.

RM48 000

State the advantages and disadvantages of the vehicle loan chosen by Ameera.

Solution:

Advantages	Disadvantages
• Repayment of vehicle loan in monthly instalments allows Ameera to own the car. • Does not require a lump sum payment.	• The car will be repossessed if instalments are not made. • The total amount of repayment is high due to interest.

Example 22

Mr Vincent is a teacher with a monthly income of RM2 800. He decides to buy a new car to commute to work. He contacts two banks to get a loan of RM40 000. In addition, every month he also needs RM1 500 to cover other expenses.

The following are loan packages offered by two banks to Mr Vincent.

Loan details	Bank A	Bank B
Loan amount	RM40 000	RM40 000
Payment period	9 years	6 years
Interest rate	4.5 %	5 %
Guarantor	Not required	Required

TIPS

Do not borrow money from unlicensed moneylenders as the loan:
♦ will be made according to its own terms and conditions
♦ imposes very high interest charge with daily compound.
♦ expose you and your family to danger if you make late payment.
♦ forces you to make additional loan to repay previous loan.

Suggest to Mr Vincent which bank is best suited for his car loan. State your reasons.

Solution:

Understanding the problem

The amount of monthly instalments payable by Mr Vincent provided that it is not burdensome.

Planning a strategy

• Calculate monthly interest.
• Calculate monthly instalment payable.

Implementing the strategy

The amount of money repaid to Bank A	The amount of money repaid to Bank B
$A = P + Prt$ $A = \text{RM40 000} + \text{RM 40 000} \times \dfrac{4.5}{100} \times 9$ Total money = RM40 000 + RM16 200 repaid = RM 56 200 Monthly instalment $= \dfrac{\text{RM56 200}}{108 \text{ months}}$ $= \text{RM520.37}$	$A = P + Prt$ $A = \text{RM40 000} + \text{RM40 000} \times \dfrac{5}{100} \times 6$ Total money = RM40 000 + RM12 000 repaid = RM52 000 Monthly instalment $= \dfrac{\text{RM52 000}}{72 \text{ months}}$ $= \text{RM722.22}$

Making a conclusion

Mr Vincent should choose Bank A because Bank A charges lower interest compared to Bank B. However, different payment term results in different amount of interest paid. Therefore, Mr Vincent could also choose Bank B.

Dynamic Challenge

Test Yourself

1. What is savings?
2. Specify features related to Fixed Deposit Account..
3. Encik Lipong deposits a sum of RM8 000 into Bank Pantas with an interest rate of 4% over 2 years. What is the amount of savings at the end of the second year?

Skills Enhancement

1. How can cost averaging strategy help an investor?
2. Explain the meaning of investment in real estate.
3. The following conversation is between Ramesh and Ismail regarding the purchase of shares.

Ramesh, I have just bought shares of BHP Berhad.

That's great, you will get returns from your investment.

Explain three types of return that will be received by Ismail.

4. The following are two types of investments.

Lee Chong bought 3 000 units of shares of a public limited company.	Mokhtar bought 3 000 000 units of unit trusts.

Explain the two differences between the two types of investments above.

5. Encik Shah wants to deposit RM10 000 into a fixed deposit account for 9 months. The following are the fixed deposit interest rates for different terms offered by a bank to Encik Shah.

Duration	Annual interest rate
1 month	3.0
3 months	3.5
6 months	3.75
9 months	4.00
12 months	4.25

Calculate the amount of interest that will be received by Encik Shah if he is saving for 9-month term.

6. In 2018, Encik Zainal holds 6 000 units of shares of Syarikat Vision Sdn. Bhd. which is worth RM1 per share unit. During the year, the company declared a 6% dividend and a bonus issue at 1 new share for 2 share units held. At the end of 2018, the share price rose to RM2.30 per unit. Calculate
(a) the amount of dividend received by Encik Zainal.
(b) the number of bonus share units to be received by Encik Zainal.
(c) the number of share units held by Encik Zainal after receiving the bonus shares.

7. Complete the following table.

Deposit amount (RM)	Flat interest rate	Savings period (years)	Total interest accumulated
10 000	5%	2	
5 000		1	150
4 000	6%		720

8. Mr Kishendran deposits RM5 000 into a fixed deposit account with 4% interest rate compounded every 3 months for a period of 3 years. Calculate the amount of interest accrued after the third year.

Self Mastery
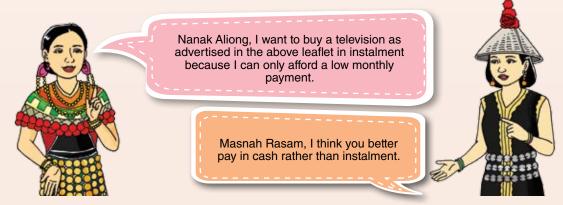
1. Mr Oswald Alphonsus borrowed RM15 000 from Bank Yakin to start a tailoring business in Rawang. The bank charges a 5% flat interest rate for a repayment period of 5 years. How much interest will be paid to the bank by Mr Oswald Alphonsus?

2. Mrs Emily Francis saves RM10 000 in a bank. By the end of the eighth year, the money collected amounts to RM19 992.71. If the bank pays an annual interest of x% for a year and is compounded every 6 months, calculate the value of x.

3. Puan Noraini Mitis deposits a certain amount of money into her savings account which provides an interest rate of 2% per annum and compounded quarterly. What is the initial deposit made by Puan Noraini Mitis if the money collected at the end of the fifth year is RM7 734.26?

4. Puan Zaiton bought 1 000 share units of Syarikat Pelita Berhad at RM2.00 per unit. At the end of the year, Syarikat Pelita Berhad paid a dividend of 20 sen per unit to all its shareholders. The following year, Puan Zaiton sold all the shares held when the share price rose to RM2.20 per unit. Calculate the return on investment for Puan Zaiton.

5. Encik Iskandar takes a personal loan of RM20 000 from Bank Cergas with an interest rate of 4% per annum. The repayment term is for 10 years. What is the monthly instalment payable by Encik Iskandar?

6. Puan Balkis takes a personal loan of RM8 000 from Bank Sentosa with interest rate of 4% per annum on the balance. The payback period is 4 years while the monthly instalment is RM110. Calculate the amount of interest payable by Puan Balkis within 2 months.

7. The following is a promotional leaflet offered by Seng Hong Company.

The following conversation took place between Masnah Rasam and Nanak Aliong after they studied the promotional leaflet above.

> Nanak Aliong, I want to buy a television as advertised in the above leaflet in instalment because I can only afford a low monthly payment.

> Masnah Rasam, I think you better pay in cash rather than instalment.

   (a) What is your view on Masnah Rasam's opinion?
   (b) Calculate the amount of interest paid and the interest rate on this instalment payments.
   (c) If you want to buy a television, how would you purchase if?

8. Ms Kayal borrows from Bank Desa RMX with an interest rate of 5% per annum. The payback period is 8 years. If the monthly instalment paid is RM218.75, calculate the amount of money borrowed by Ms Kayal.

9. Mr Murugan has borrowed RM16 000 from Bank Orkid for personal use. He will repay over 5 years with a monthly instalment of RM320. Calculate the yearly interest charged by the bank.

10. Puan Sapiah borrowed RM12 000 from a bank with interest rate of 3% per annum for 5 years. Meanwhile, Puan Shafiqah Ira borrows the same amount of money from another bank with a rate of 4.5% per annum for 5 years. Calculate and state the difference between the total interest paid by Puan Sapiah and Puan Shafiqah Ira.

PROJECT 📋
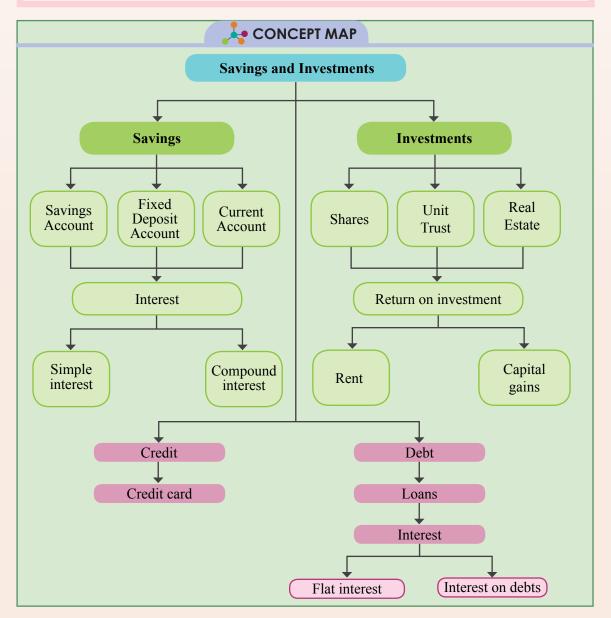
Assume that you have won RM1 million in a puzzle contest.

1. State the way in which you will invest the money.

2. Explain why you chose this way of investment.

Buy Yakinz products and stand a chance to win prizes worth over RM1 000 000*

Contest
SMS & WIN

Contest Period: 1 September – 31 October 2018

SPECIMEN

Terms and conditions apply. For more information, please visit www.yakinz.com.my

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🔬 CONCEPT MAP

Savings and Investments

Savings

- Savings Account
- Fixed Deposit Account
- Current Account

Interest
- Simple interest
- Compound interest

Credit
- Credit card

Investments

- Shares
- Unit Trust
- Real Estate

Return on investment
- Rent
- Capital gains

Debt
- Loans
- Interest
  - Flat interest
  - Interest on debts

SELF-REFLECT

At the end of this chapter, I can:

		🌻😊	🌻☹️
1.	Recognise various types of savings and investments.		
2.	Perform calculations involving simple interest and compound interest for savings, and hence explain the impact of changes in period, rate of interest or return and compounding frequency on the future value of savings.		
3.	Perform calculations involving return on investments, and hence explain the factors that affect return on investments and its impacts.		
4.	Compare and contrast potential risks, return and liquidity of various types of savings and investments.		
5.	Calculate the average cost per share for the investment of shares using the ringgit cost averaging strategy and explain the benefits of the strategy.		
6.	Solve problems involving savings and investments.		
7.	Explain the meaning of credit and debt, and hence describe the wise management of credit and debt.		
8.	Investigate and describe the advantages and disadvantages of credit card and ways to use it wisely.		
9.	Investigate and describe the impact of minimum and late payments for credit card usage.		
10.	Solve problems involving the use of credit cards.		
11.	Calculate the total amount of loan repayment and instalment, with various interest rates and different loan periods.		
12.	Solve problems involving loans.		

CHAPTER 1

⚙️ EXPLORING MATHEMATICS

You can visit the Credit Counselling and Debt Management Agency (AKPK) website to calculate the required period and the amount of interest payable to settle your credit card debt.

CHAPTER 4
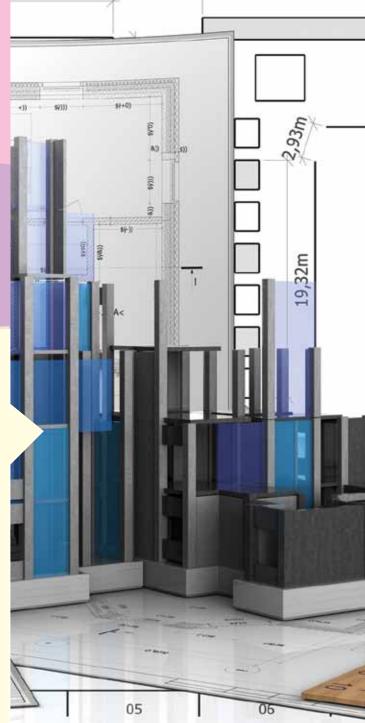
Scale Drawings

Why do you learn this chapter?

- Scale drawings are used to give an idea of object measurements or actual distances.
- Scale drawings are frequently used in architecture, engineering, photography, technological design, and so on.
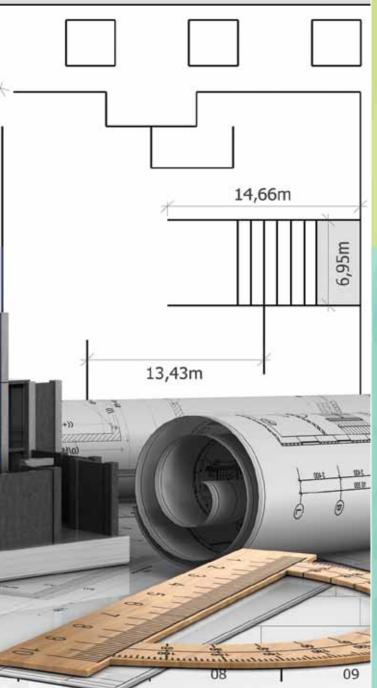
Housing developers usually prepare models of the housing scheme they are going to build. These models give the buyers an idea of the housing zone and other facilities provided by the developer.

Also, the plans of the houses to be built are drawn using a certain scale with measurements being in proportion to the actual measurements of the houses. These plan drawings allow buyers to choose the type of house to buy based on house size, facilities and also needs and affordability. Have you ever seen your house plan?

Angkor Wat in Cambodia is one of the famous monuments in Southeast Asia. It was built by Suryavarman II after his victory over Champa and subsequently uniting Kampuchea. This monument symbolises the strength and sovereignty of Suryavarman II's government. Its height is 213 metres and its area is 208 hectares. The altar is at the centre. It has four storeys with every corner having walls measuring 850 metres wide and 1000 metres long built from laterite and sandstones found outside. Carvings depicting stories from the epic Ramayana and Mahabharata decorate the walls.

http://bukutekskssm.my/Mathematics/F3/ExploringEraChapter4.pdf

14,66m

6,95m

13,43m

WORD BANK

• original	• *asal*
• geometrical shape	• *bentuk geometri*
• degree	• *darjah*
• grid	• *grid*
• sketch	• *lakar*
• object	• *objek*
• size	• *saiz*
• scale	• *skala*

4.1 Scale Drawings

✏️ **What is the relationship between the actual measurements and the measurements of various sizes of drawings of an object?**

Do you know that the maps found in navigation software are drawn to a certain scale?

The distance between two towns shown in the software is proportional to the actual distance.
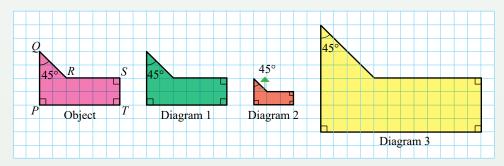
| 20 mi
| 50 km

For example, in the picture above, the distance between Johor Bahru and Kuala Lumpur is shown using a scale of 1 cm : 50 km.

Example 1

The diagram below shows the drawings representing object *PQRST* drawn to different sizes.

What can you say about the size of Diagram 1, Diagram 2 and Diagram 3 compared to object *PQRST*?

Solution:

Diagram 1: Lengths of all sides and sizes of all angles are the same as object.

Diagram 2: Lengths of all sides reduced by a certain proportion compared to object but sizes of all angles unchanged.

Diagram 3: Lengths of all sides increased by a certain proportion compared to object but sizes of all angles unchanged.

In conclusion, all sides of Diagram 1, Diagram 2 and Diagram 3 follow a certain scale that is proportional to the object whereas angle size remains unchanged. Therefore, Diagram 1, Diagram 2 and Diagram 3 are scale drawings of object *PQRST*.
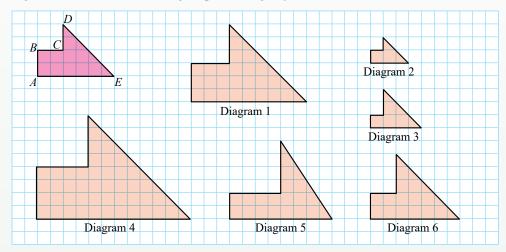
Scale drawing is the drawing of an object with all measurements in the drawing proportional to the measurements of the object.

MIND TEST 4.1a

1. The diagram below shows drawings representing object *ABCDE* drawn to different sizes.

Diagram 1
Diagram 2
Diagram 3
Diagram 4
Diagram 5
Diagram 6

State the diagram which is the scale drawing of object *ABCDE*.

2. Using grid paper, draw all the shapes below using
   (a) the same size   (b) smaller size   (c) bigger size

(i)   (ii)   (iii)   (iv)

✎ How do you interpret the scale of a scale drawing?
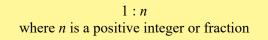
The scale used to draw a scale drawing depends on the ratio of measurement of scale drawing to measurement of object, which is

$$\text{Scale} = \frac{\text{Measurement of scale drawing}}{\text{Measurement of object}}$$

This ratio can also be written in the form;

Measurement of scale drawing : Measurement of object

Usually, for scale drawings, we use scale in the form of ratio.

$1 : n$
where n is a positive integer or fraction

$1 : n$ means one unit on the scale drawing will represent n units on the object.

Brainstorming 1 👀 👥 In pairs

Aim: Interpret the scale of a scale drawing.

Steps:

1. Study the diagrams below.

2. Complete the table below based on the diagrams above.

Diagram	Scale drawing Side	Scale drawing Length (unit)	Object Side	Object Length (unit)	Scale Ratio	Scale 1 : n
Diagram 1	$P'Q'$	12	PQ	4	12 : 4	$1 : \frac{1}{3}$
	$P'R'$	24	PR	8	24 : 8	$1 : \frac{1}{3}$
Diagram 2	$P'Q'$		PQ			
	$P'R'$		PR			
Diagram 3	$P'Q'$		PQ			
	$P'R'$		PR			
Diagram 4	$P'Q'$		PQ			
	$P'R'$		PR			
Diagram 5	$P'Q'$		PQ			
	$P'R'$		PR			
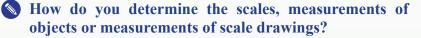
Discussion:
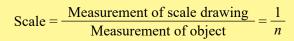
Discuss the results based on the table above.

From Brainstorming 1, it is found that;

> If $n < 1$, then the size of the scale drawing is bigger than the size of the object.
> If $n > 1$, then the size of the scale drawing is smaller than the size of the object.
> If $n = 1$, then the size of the scale drawing is the same as the size of the object.
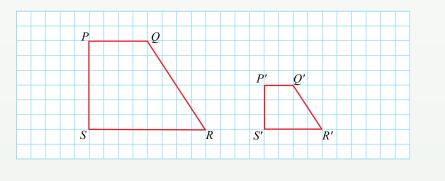
✏️ **How do you determine the scales, measurements of objects or measurements of scale drawings?**

$$\text{Scale} = \frac{\text{Measurement of scale drawing}}{\text{Measurement of object}} = \frac{1}{n}$$

Example 2

The diagram below shows object $PQRS$ and scale drawing $P'Q'R'S'$ drawn on a grid of equal squares. State the scale used in the form $1 : n$.

Solution:

$$\text{Scale} = \frac{P'Q'}{PQ} = \frac{2}{4} = \frac{1}{2} \qquad \text{or} \qquad \text{Scale} = \frac{P'S'}{PS} = \frac{3}{6} = \frac{1}{2} \qquad \text{Thus, scale} = 1 : 2$$
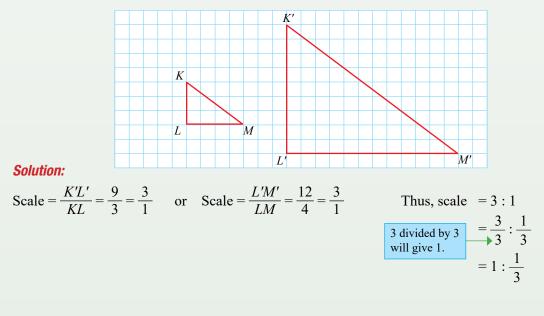
Example 3
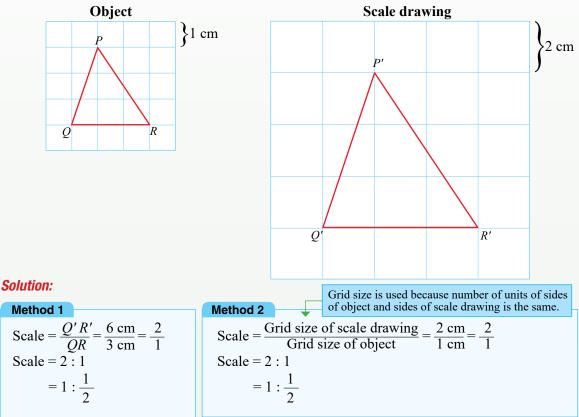
The diagram below shows object KLM and scale drawing $K'L'M'$ drawn on a grid of equal squares. State the scale used in the form $1 : n$.

Solution:

$$\text{Scale} = \frac{K'L'}{KL} = \frac{9}{3} = \frac{3}{1} \qquad \text{or} \qquad \text{Scale} = \frac{L'M'}{LM} = \frac{12}{4} = \frac{3}{1} \qquad \text{Thus, scale} = 3 : 1$$

3 divided by 3 will give 1.

$$= \frac{3}{3} : \frac{1}{3}$$
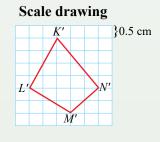$$= 1 : \frac{1}{3}$$

CHAPTER 4
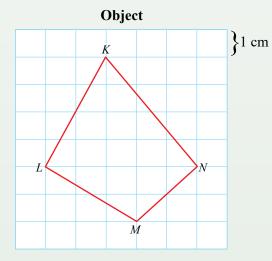
Example 4

The diagram below shows object *PQR* and scale drawing *P'Q'R'* drawn on a grid of equal squares of different sizes. Determine the scale used in the form 1 : *n*.
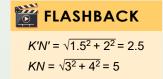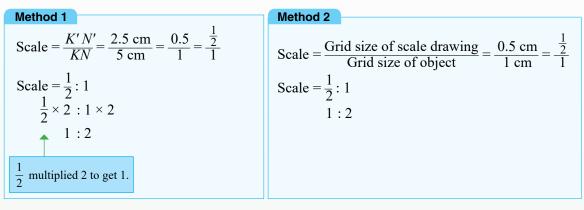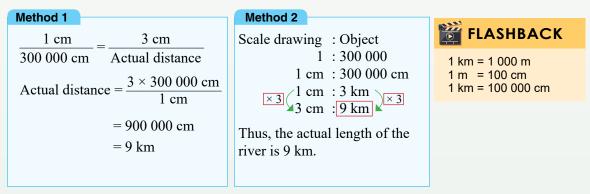
Object

} 1 cm

Scale drawing

} 2 cm

Solution:

> Grid size is used because number of units of sides of object and sides of scale drawing is the same.

Method 1

$\text{Scale} = \dfrac{Q'R'}{QR} = \dfrac{6 \text{ cm}}{3 \text{ cm}} = \dfrac{2}{1}$

$\text{Scale} = 2 : 1$

$= 1 : \dfrac{1}{2}$

Method 2

$\text{Scale} = \dfrac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \dfrac{2 \text{ cm}}{1 \text{ cm}} = \dfrac{2}{1}$

$\text{Scale} = 2 : 1$

$= 1 : \dfrac{1}{2}$

Example 5

The diagram below shows object *KLMN* and scale drawing *K'L'M'N'* drawn on a grid of equal squares of different sizes. Determine the scale used.

Object

} 1 cm

Scale drawing

} 0.5 cm

FLASHBACK

$K'N' = \sqrt{1.5^2 + 2^2} = 2.5$

$KN = \sqrt{3^2 + 4^2} = 5$

Solution:

Method 1

Scale $= \dfrac{K'N'}{KN} = \dfrac{2.5 \text{ cm}}{5 \text{ cm}} = \dfrac{0.5}{1} = \dfrac{\frac{1}{2}}{1}$

Scale $= \dfrac{1}{2} : 1$

$\dfrac{1}{2} \times 2 : 1 \times 2$

$1 : 2$

$\dfrac{1}{2}$ multiplied 2 to get 1.

Method 2

Scale $= \dfrac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \dfrac{0.5 \text{ cm}}{1 \text{ cm}} = \dfrac{\frac{1}{2}}{1}$

Scale $= \dfrac{1}{2} : 1$

$1 : 2$

Example 6

A map is drawn to a scale of 1 : 300 000. Calculate the actual length, in km, of a river that is 3 cm long on the map.

Solution:

Method 1

$\dfrac{1 \text{ cm}}{300\,000 \text{ cm}} = \dfrac{3 \text{ cm}}{\text{Actual distance}}$

Actual distance $= \dfrac{3 \times 300\,000 \text{ cm}}{1 \text{ cm}}$

$= 900\,000 \text{ cm}$

$= 9 \text{ km}$

Method 2

Scale drawing : Object

$1 : 300\,000$

$1 \text{ cm} : 300\,000 \text{ cm}$

$\boxed{\times 3} \big(1 \text{ cm} : 3 \text{ km} \big) \boxed{\times 3}$

$3 \text{ cm} : \boxed{9 \text{ km}}$

Thus, the actual length of the river is 9 km.

FLASHBACK

1 km = 1 000 m
1 m = 100 cm
1 km = 100 000 cm
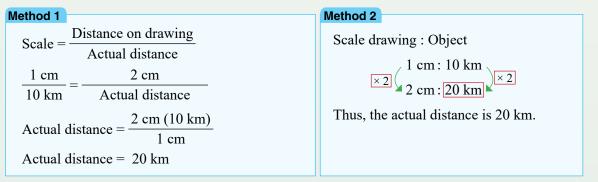
Example 7

The map of Johor is drawn to a scale of 1 cm to 10 km. Calculate the actual distance between Kluang and Ayer Hitam if the distance on the map is 2 cm.

Solution:

Method 1

Scale $= \dfrac{\text{Distance on drawing}}{\text{Actual distance}}$

$\dfrac{1 \text{ cm}}{10 \text{ km}} = \dfrac{2 \text{ cm}}{\text{Actual distance}}$

Actual distance $= \dfrac{2 \text{ cm} (10 \text{ km})}{1 \text{ cm}}$

Actual distance $= 20 \text{ km}$

Method 2

Scale drawing : Object

$\boxed{\times 2} \big(1 \text{ cm} : 10 \text{ km} \big) \boxed{\times 2}$

$2 \text{ cm} : \boxed{20 \text{ km}}$

Thus, the actual distance is 20 km.
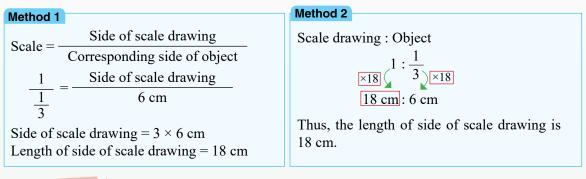
Example 8
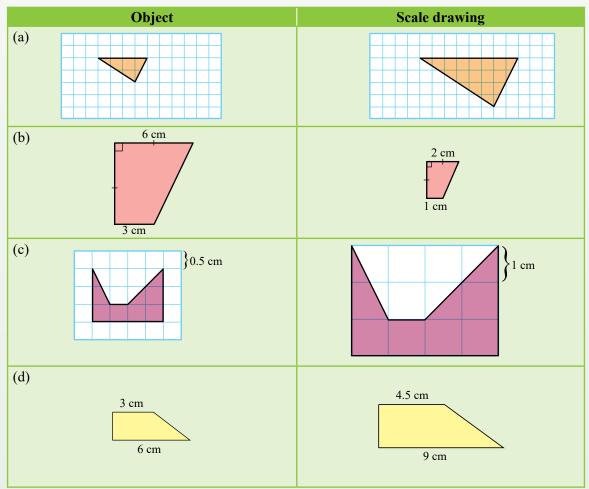
Khairul draws a square to a scale of $1 : \dfrac{1}{3}$. If the actual length of sides of the square is 6 cm, what is the length of sides, in cm, of the scale drawing?

Solution:

Method 1

$$\text{Scale} = \frac{\text{Side of scale drawing}}{\text{Corresponding side of object}}$$

$$\frac{1}{\frac{1}{3}} = \frac{\text{Side of scale drawing}}{6 \text{ cm}}$$

Side of scale drawing = 3 × 6 cm
Length of side of scale drawing = 18 cm

Method 2

Scale drawing : Object

$$1 : \frac{1}{3}$$

×18 ↘ ↙ ×18

18 cm : 6 cm

Thus, the length of side of scale drawing is 18 cm.

MIND TEST 4.1b

1. Determine the scale used for each scale drawing below in the form $1 : n$.

Object	Scale drawing
(a)	
(b) 6 cm, 3 cm	2 cm, 1 cm
(c) 0.5 cm	1 cm
(d) 3 cm, 6 cm	4.5 cm, 9 cm

2. A poster has a length of 24 cm and a width of 8 cm. Calculate the length and width of the scale drawing of the poster, in cm, that is drawn to a scale of 1 : 4.

3. A map is drawn to a scale of 1 : 400 000. What is the actual length, in km, of a river with a length of 2.5 cm on the map?

4. Siew Lin draws a right-angled triangle to a scale of $1 : \dfrac{1}{3}$. If the hypotenuse of the scale drawing is 18 cm, calculate the length of the hypotenuse of the original triangle.

✎ How do you draw the scale drawings of objects and vice versa?

🔢 Drawing the scale drawing of an object.

There are three ways to draw the scale drawing of an object.
(a) Use grid paper of the same size for different scales or;
(b) Use grid paper of different sizes or;
(c) Draw on a blank paper according to the given scale.

Example 9

Draw the scale drawing of shape *PQRS* on a grid of equal squares using a scale of $1 : \dfrac{1}{2}$.
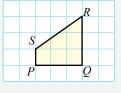
Solution:

The scale given is $1 : \dfrac{1}{2}$. Therefore, every side of the scale drawing is two times longer than the length of sides of object *PQRS*.
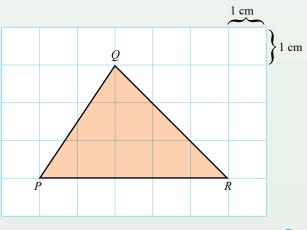
QUIZ
If you have to draw the scale drawing of your school field, what is the suitable scale to be used? Why?

Example 10
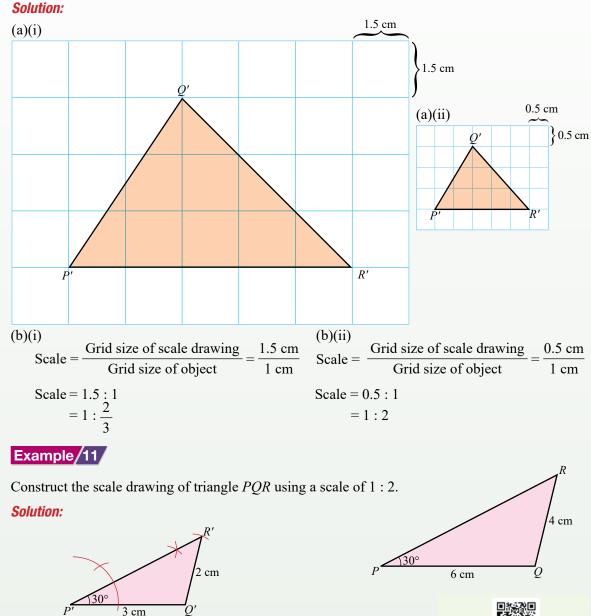
(a) Diagram △*PQR* is drawn on a grid of 1 cm × 1 cm. Redraw △*PQR* on grid paper with dimensions
  (i) 1.5 cm × 1.5 cm
  (ii) 0.5 cm × 0.5 cm
(b) Calculate the scale used in (a)(i) and (a)(ii) in the form 1 : *n*.

95
KPM

Solution:

(a)(i)

1.5 cm

1.5 cm

(a)(ii)

0.5 cm

0.5 cm

(b)(i)

$$\text{Scale} = \frac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \frac{1.5 \text{ cm}}{1 \text{ cm}}$$

$$\text{Scale} = 1.5 : 1$$
$$= 1 : \frac{2}{3}$$

(b)(ii)

$$\text{Scale} = \frac{\text{Grid size of scale drawing}}{\text{Grid size of object}} = \frac{0.5 \text{ cm}}{1 \text{ cm}}$$

$$\text{Scale} = 0.5 : 1$$
$$= 1 : 2$$

Example 11

Construct the scale drawing of triangle PQR using a scale of 1 : 2.

Solution:

For objects with given angles, the angles of the scale drawing must be accurately drawn and the lengths of sides are drawn to scale.
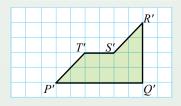
🖩 **Drawing the objects for a scale drawing.**

Example 12

The diagram shows a scale drawing drawn on a grid of equal squares to a scale of 1 : 2. Draw the actual object for $P'Q'R'S'T'$.
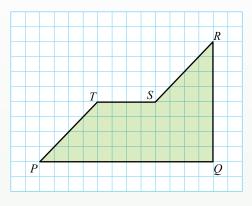
KPM

Solution:
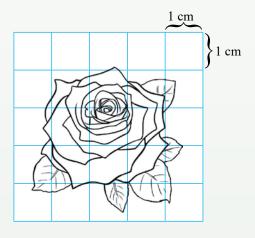
The scale used is 1 : 2, that is the size of scale drawing is two times smaller than the object. Therefore, every side of actual object is two times longer than the sides of scale drawing.

Example/13
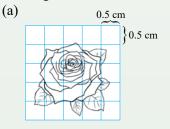
The diagram shows the scale drawing of a flower drawn on 1 cm × 1 cm grids. Draw the actual object on grids of
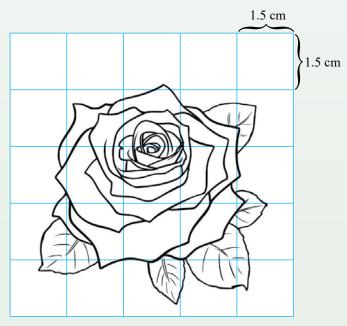
(a) 0.5 cm × 0.5 cm

(b) 1.5 cm × 1.5 cm

Solution:

Object must be drawn on grids of different sizes. Thus, the number of units of sides of object is the same as the number of units of sides of scale drawing.

(a)

97

KPM
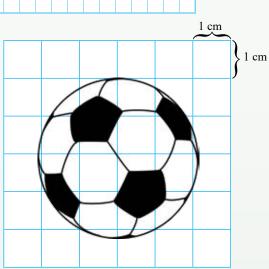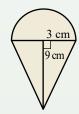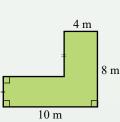
1. Draw the scale drawing of each object below to a scale of $1 : \dfrac{1}{2}$ and $1 : 3$.

(a)

(b)

2. (a) The object in the diagram is drawn on 1 cm × 1 cm grid paper. Redraw the shape of the object on a grid paper of

   (i) 2 cm × 2 cm

   (ii) 0.5 cm × 0.5 cm

   (b) Calculate the scale used in (a)(i) and (a)(ii).

1 cm

1 cm
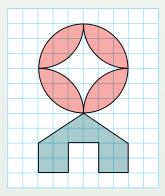
3. Draw the scale drawing of the following shapes to the given scale.

   (a) Scale 1 : 3

   3 cm
   9 cm

   (b) Scale 1 : 200

   4 m
   8 m
   10 m

   (c) Scale $1 : \dfrac{1}{2}$

   4 cm
   2 cm

4. The diagram shows the scale drawing of a composite shape that is drawn on a grid of equal squares to a scale of $1 : \dfrac{1}{2}$.

   Draw the actual object for the shape.

✎ How do you solve problems involving scale drawings?

Example 14

The distance on a map between Bintulu and Miri is 4 cm.

(a) If the scale used to draw the map is 1 cm : 50 km, calculate the actual distance, in km, between Bintulu and Miri.

(b) If the map is redrawn to a scale of 1 : 2 000 000, calculate the distance between Bintulu and Miri on the new map.

(c) Mr Dominic Lajawa and family wants to visit Miri. If he plans to drive to Miri at a speed of 80 kmh^{-1}, calculate the time taken to drive from Bintulu to Miri in hours and minutes.

TIP 💡

If the scale of scale drawing and requirement of question are in the same unit, the scale need not be changed to cm.

Solution:

Understanding the problem

- Actual distance for 4 cm drawn to scale of 1 cm : 50 km.

- Distance on scale drawing drawn to scale of 1 : 2 000 000.

- Time in hours and minutes for journey from Bintulu to Miri at speed of 80 kmh^{-1}.
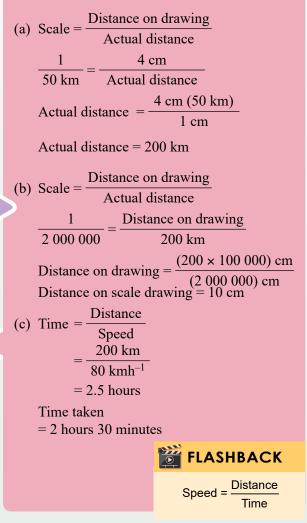
Planning a strategy

$$\text{Scale} = \frac{\text{Distance on drawing}}{\text{Actual distance}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Making a conclusion

- Actual distance between Bintulu and Miri is 200 km.

- Distance between Bintulu and Miri on the map of scale of 1 : 2 000 000.

- Time taken for Mr Dominic Lajawa to drive from Bintulu to Miri at a speed of 80 kmh^{-1} is 2 hours 30 minutes.
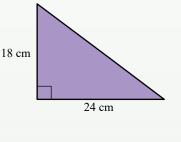
Implementing the strategy

(a) $\text{Scale} = \dfrac{\text{Distance on drawing}}{\text{Actual distance}}$

$\dfrac{1}{50 \text{ km}} = \dfrac{4 \text{ cm}}{\text{Actual distance}}$

$\text{Actual distance} = \dfrac{4 \text{ cm } (50 \text{ km})}{1 \text{ cm}}$

$\text{Actual distance} = 200 \text{ km}$

(b) $\text{Scale} = \dfrac{\text{Distance on drawing}}{\text{Actual distance}}$

$\dfrac{1}{2\,000\,000} = \dfrac{\text{Distance on drawing}}{200 \text{ km}}$

$\text{Distance on drawing} = \dfrac{(200 \times 100\,000) \text{ cm}}{(2\,000\,000) \text{ cm}}$

$\text{Distance on scale drawing} = 10 \text{ cm}$

(c) $\text{Time} = \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{200 \text{ km}}{80 \text{ kmh}^{-1}}$

$= 2.5 \text{ hours}$

Time taken

$= 2 \text{ hours } 30 \text{ minutes}$

🎬 **FLASHBACK**

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

1. The diagram shows a right-angled triangle. A scale drawing of the triangle is drawn to a scale of $1 : \frac{1}{3}$. Calculate the area, in cm², of the scale drawing.
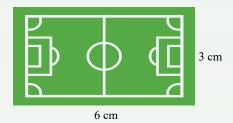
18 cm

24 cm

2. The diagram shows a room in the shape of a rectangle. Calculate the perimeter, in cm, of the scale drawing of the room which is drawn to a scale of 1 : 50.

3.5 m

5.2 m

3. The measurements of a rectangular room on a scale drawing are 7 cm × 5 cm. If the scale used is 1 : 400, calculate the actual area of the room in m².

4. A regular polygon with exterior angle of 36° is redrawn using a scale of 1 : 5. If the actual length of sides of the regular polygon is 10 cm, calculate the perimeter of the scale drawing of the regular polygon.

5.

3 cm

6 cm

The diagram above shows a scale drawing of a rectangular field.
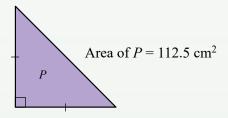
(a) If the scale used is 1 : 2 000, calculate the actual area of the field in square metres.

(b) Mr Dany cuts the grass on the field at a rate of 400 square metres in 8 minutes. Calculate the time, in hours and minutes, that Mr Dany takes to cut all the grass on the field.
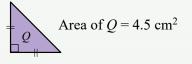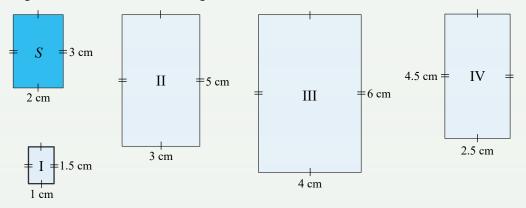
Dynamic Challenge

Test Yourself

1. The diagram below shows triangle P which is the scale drawing of triangle Q with a scale of $1 : n$. Calculate the value of n.
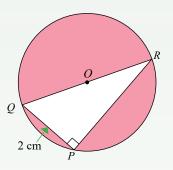
Area of P = 112.5 cm²

Area of Q = 4.5 cm²

2. The diagram below shows five rectangles.

S ⊨3 cm

2 cm

I ⊨1.5 cm

1 cm

II ⊨5 cm

3 cm

III ⊨6 cm

4 cm

4.5 cm⊨ IV

2.5 cm

(a) Among rectangles I, II, III and IV, which are the scale drawings of rectangle S drawn to a certain scale?

(b) For each answer in (a), determine the scale used.

(c) (i) Calculate the area of each rectangle, in cm², for your answer in (a).

    (ii) Determine the ratio of area of S to area of each answer in (c)(i).

    What are your conclusions about the ratios obtained?

3. The diagram shows a scale drawing of a circle with centre O and triangle PQR. Given that diameter of circle is 6 cm and the scale of the drawing is 1 : 3.

(a) Calculate the actual length of PR in cm. State your answer correct to 3 significant figures.

(b) Using your answer in (a), calculate the actual area of the shaded region in cm². State the answer correct to 4 significant figures.

R

O
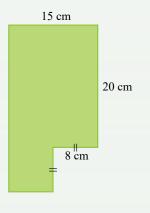
Q

2 cm

P

CHAPTER 4

1.

The distance by air from Kuching to Kota Kinabalu on a map is 5.4 cm. Given the scale of the map is 1 cm : 150 km. If an aeroplane takes off from Kuching International Airport at 1240 hours and lands at Kota Kinabalu International Airport at 1410 hours, calculate the average speed of the aeroplane in kmh^{-1}.

2. The diagram shows the scale drawing of Puan Farah's living room. The scale of the drawing is 1 : 50. Puan Farah wants to lay tiles throughout the entire living room. She intends to use tiles measuring 30 cm × 30 cm which cost RM2.80 a piece. Puan Farah's husband suggests to use tiles of 50 cm × 50 cm at RM6 a piece. Which tile should Puan Farah choose if she wants to save money? State the reason for your answer.

3. The diagram shows the scale drawing of a rectangular farm owned by Pak Hassan. Given the scale of the drawing is 1 : 2 000.
   (a) Calculate the actual area of the freshwater fish pond to the nearest square metre.
   $$\left[\pi = \frac{22}{7}\right].$$
   (b) Calculate the ratio of the area planted with durian trees to the area planted with banana trees.
   (c) Calculate the area, in m^2, of the vacant land.
   (d) Pak Hassan wants to fence up his farm. If the cost of one metre of fencing is RM5.50, calculate the total cost of fencing, in RM.

Self Mastery

1. The diagram shows the scale drawing of the floor plan of a shophouse that is drawn to a scale of 1 : 400.

   (a) Calculate the actual area of the storeroom, in m^2.

   (b) State the ratio of the area of the shophouse to the area of the storeroom.

   (c) If the actual height of the shophouse is 3.75 m, calculate the volume, in m^3, of the three-dimensional shophouse.
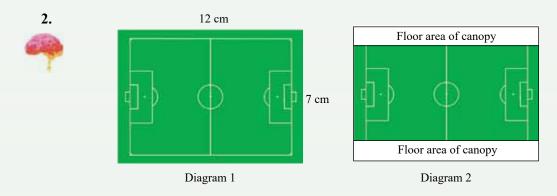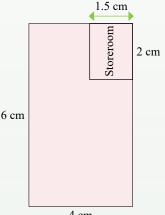
2.

Diagram 1   Diagram 2

Diagram 1 shows the scale drawing of a rectangular football field.

   (a) If this scale drawing is drawn to a scale of 1 : 1 000, calculate the actual area, in m^2, of the football field.

   (b) Sharon wants to redraw the scale drawing in Diagram 1 on a piece of A4-sized paper. What is the maximum scale that Sharon can choose? State the reasons for your answer.

   (c) Several canopies will be set up on the football field as in Diagram 2 for a carnival.

      (i) If the dimensions of the base of a tent are 5 m × 4 m, calculate the maximum number of tents that can be erected.

      (ii) The rent for a tent is RM100 a day. A 25% discount will be given if the tent is rented for five days or more. Calculate the total rent, in RM, if the carnival lasts for one week.
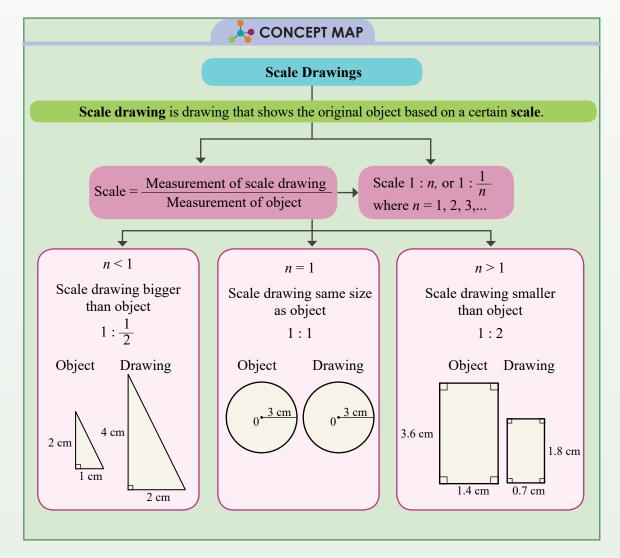
CHAPTER 4

PROJECT 🗒️

Draw the map of the district where you live using a suitable scale. You can mark the location of your house, school and interesting places in your district using symbols or suitable illustrations. Exhibit your project in the classroom.
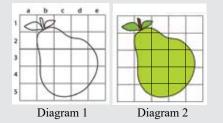

🧬 **CONCEPT MAP**

Scale Drawings

Scale drawing is drawing that shows the original object based on a certain **scale**.

$$\text{Scale} = \frac{\text{Measurement of scale drawing}}{\text{Measurement of object}}$$

Scale $1 : n$, or $1 : \dfrac{1}{n}$
where $n = 1, 2, 3,...$

$n < 1$	$n = 1$	$n > 1$
Scale drawing bigger than object	Scale drawing same size as object	Scale drawing smaller than object
$1 : \dfrac{1}{2}$	$1 : 1$	$1 : 2$

Object   Drawing

2 cm, 4 cm, 1 cm, 2 cm

Object   Drawing

3 cm, 3 cm

Object   Drawing

3.6 cm, 1.8 cm, 1.4 cm, 0.7 cm

(SELF-REFLECT)

At the end of this chapter, I can:

		🌻	🥀
1.	Investigate and explain the relationship between the actual measurements and the measurements of various sizes of drawings of an object, and hence explain the meaning of scale drawing.		
2.	Interpret the scale of a scale drawing.		
3.	Determine the scales, measurements of objects or measurements of scale drawings.		
4.	Draw the scale drawings of objects and vice versa.		
5.	Solve problems involving scale drawings.		

⚙ EXPLORING MATHEMATICS

1. Download grid paper of various sizes.

2. Draw your favourite object as shown in Diagram 1 or Diagram 2 on one of the grid papers chosen.

Scan the QR Code or visit http://bukutekskssm. my/Mathematics/F3/ Chapter4Grid.pdf to download grid paper of various sizes.

Diagram 1   Diagram 2

3. Redraw the drawing on all the grid papers of different sizes.

4. Can you easily draw your favourite object on grids of different sizes?

5. Exhibit your work at the mathematics corner of your classroom.

CHAPTER 4

Answers 📚

CHAPTER 1 Indices

1.

Base	Index
5	3
−4	7
$\frac{1}{2}$	10
m	6
n	0
0.2	9
$-\frac{3}{7}$	4
x	20
$2\frac{1}{3}$	2
8	1

2. (a) 6^6

(b) $(0.5)^7$

(c) $\left(\frac{1}{2}\right)^4$

(d) $(-m)^5$

(e) $\left(1\frac{2}{3}\right)^3$

(f) $\left(-\frac{1}{n}\right)^6$

3. (a) $(-3) \times (-3) \times (-3)$

(b) $2.5 \times 2.5 \times 2.5 \times 2.5$

(c) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

(d) $(-2\frac{1}{4}) \times (-2\frac{1}{4}) \times (-2\frac{1}{4})$

(e) $k \times k \times k \times k \times k \times k$

(f) $(-p) \times (-p) \times (-p) \times (-p) \times (-p) \times (-p) \times (-p)$

(g) $\frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m} \times \frac{1}{m}$

(h) $(3n) \times (3n) \times (3n) \times (3n) \times (3n)$

1. (a) 3^4 (b) 5^6 (c) $\left(\frac{4}{5}\right)^3$

(d) $(0.2)^5$ (e) $(-4)^7$ (f) $\left(-\frac{1}{4}\right)^2$

1. (a) 6 561 (b) −1 024 (c) 15.625

(d) −32.768 (e) $\frac{243}{32\,768}$ (f) $\frac{1}{1\,296}$

(g) $2\frac{7}{9}$ (h) $-12\frac{19}{27}$

1. (a) 3^7 (b) $(-0.4)^8$ (c) $\left(\frac{4}{7}\right)^9$

(d) $\left(-1\frac{2}{5}\right)^{10}$ (e) $-6m^9$ (f) $\frac{n^{12}}{5}$

(g) $-15x^7$ (h) y^{12}

1. (a) $5^5 \times 9^5$ (b) $(0.4)^3 \times (1.2)^9$

(c) $4x^6y^7$ (d) $-\frac{3}{2}k^6p^{11}$

1. (a) 4 (b) 7^2 (c) m^4n^5

(d) $3xy^3$ (e) m (f) $-5h$

2. (a) $8^{\boxed{8}} \div 8^4 \div 8^3 = 8$ **3.** 8

(b) $m^4n^{\boxed{6}} \div m^{\boxed{2}}n^5 = m^2n$

(c) $\dfrac{m^{10}n^4 \times m^{\boxed{2}}n^2}{m^7n} = m^5n^{\boxed{5}}$

(d) $\dfrac{27x^3y^6 \times xy^{\boxed{2}}}{\boxed{9}x^2y^3} = 3x^{\boxed{2}}y^5$

1. (a) 12^{10} (b) 3^{20} (c) 7^6 (d) $(-4)^{21}$

(e) k^{24} (f) g^{26} (g) $(-m)^{12}$ (h) $(-c)^{21}$

2. (a) True (b) False (c) False (d) False

1. (a) $2^2 \times 3^8$ (b) $11^9 \times 9^{15}$ (c) $13^6 \div 7^{12}$

(d) $5^{15} \times 3^{20}$ (e) $m^{15}n^{20}p^{10}$ (f) $16w^8x^{12}$

(g) $\dfrac{729a^{30}}{b^{24}}$ (h) $\dfrac{8a^{15}}{27b^{12}}$

2. (a) $11^2 \times 4^4$ (b) $3^3 \times 6^2$ (c) $\dfrac{4^4}{6^6}$

(d) $(-4)^6 \times (-5)^4$ (e) x^4y^4

(f) $h^{10}k^6$ (g) $m^{11}n^{15}$ (h) b^2d^6

3. (a) $6mn^8$ (b) $10x^8y^3$ (c) de

1. (a) $\dfrac{1}{5^3}$ (b) $\dfrac{1}{8^4}$ (c) $\dfrac{1}{x^8}$ (d) $\dfrac{1}{y^{16}}$

(e) a^4 (f) 20^2 (g) $\dfrac{3}{n^4}$ (h) $-\dfrac{5}{n^6}$

(i) $\dfrac{2}{7m^5}$ (j) $-\dfrac{3}{8m^4}$ (k) $\left(\dfrac{5}{2}\right)^{12}$ (l) $\left(-\dfrac{7}{3}\right)^{14}$

(m) $\left(\dfrac{y}{x}\right)^{10}$ (n) $\left(\dfrac{3y}{2x}\right)^4$ (o) $(2x)^5$

2. (a) 5^{-4} (b) 8^{-3} (c) m^{-7} (d) n^{-9}

(e) $\dfrac{1}{10^{-2}}$ (f) $\dfrac{1}{(-4)^{-3}}$ (g) $\dfrac{1}{m^{-12}}$ (h) $\dfrac{1}{n^{-16}}$

(i) $\left(\dfrac{7}{4}\right)^{-9}$ (j) $\left(\dfrac{y}{x}\right)^{-10}$

3. (a) $\dfrac{1}{4}$ (b) $\dfrac{2^4}{3^{14}}$ (c) $2^6 \times 5^2$

(d) $\dfrac{1}{3m^3n^7}$ (e) $\dfrac{1}{8m^8}$ (f) $\dfrac{m^6n}{18}$

1. (a) $125^{\frac{1}{3}}$ (b) $2\,187^{\frac{1}{7}}$ (c) $(-1\,024)^{\frac{1}{5}}$ (d) $n^{\frac{1}{10}}$

2. (a) $\sqrt{4}$ (b) $\sqrt[5]{32}$ (c) $\sqrt[3]{-729}$ (d) $\sqrt[15]{n}$

3. (a) 7 (b) −6 (c) 8 (d) −8

MIND TEST 1.2h

$a^{\frac{m}{n}}$	$729^{\frac{5}{6}}$	$121^{\frac{3}{2}}$	$w^{\frac{3}{7}}$	$x^{\frac{2}{5}}$	$\left(\frac{16}{81}\right)^{\frac{3}{4}}$	$\left(\frac{h}{k}\right)^{\frac{2}{3}}$
$(a^m)^{\frac{1}{n}}$	$(729^5)^{\frac{1}{6}}$	$(121^3)^{\frac{1}{2}}$	$(w^3)^{\frac{1}{7}}$	$(x^2)^{\frac{1}{5}}$	$\left[\left(\frac{16}{81}\right)^3\right]^{\frac{1}{4}}$	$\left[\left(\frac{h}{k}\right)^2\right]^{\frac{1}{3}}$
$(a^{\frac{1}{n}})^m$	$(729^{\frac{1}{6}})^5$	$(121^{\frac{1}{2}})^3$	$(w^{\frac{1}{7}})^3$	$(x^{\frac{1}{5}})^2$	$\left[\left(\frac{16}{81}\right)^{\frac{1}{4}}\right]^3$	$\left[\left(\frac{h}{k}\right)^{\frac{1}{3}}\right]^2$
$\sqrt[n]{a^m}$	$\sqrt[6]{729^5}$	$\sqrt{121^3}$	$\sqrt[7]{w^3}$	$\sqrt[5]{x^2}$	$\sqrt[4]{\left(\frac{16}{81}\right)^3}$	$\sqrt[3]{\left(\frac{h}{k}\right)^2}$
$(\sqrt[n]{a})^m$	$(\sqrt[6]{729})^5$	$(\sqrt{121})^3$	$(\sqrt[7]{w})^3$	$(\sqrt[5]{x})^2$	$\left(\sqrt[4]{\frac{16}{81}}\right)^3$	$\left(\sqrt[3]{\frac{h}{k}}\right)^2$

MIND TEST 1.2i

1. (a) 9   (b) 4   (c) 4   (d) 8
   (e) 256   (f) 16   (g) 216   (h) 343
   (i) 7   (j) 1 331   (k) 169   (l) 1 000

2. (a) $\sqrt[2]{6\ 561}^{\boxed{1}}$, $3^{\frac{4}{\boxed{}}}$, $9^{\boxed{2}}$, $81^{\boxed{1}}$, $243^{\frac{4}{5}}$, $27^{\frac{4}{3}}$
   (b) $25^{\frac{3}{\boxed{}}}$, $125^{\boxed{1}}$, $625^{\frac{3}{4}}$, $\sqrt[2]{15\ 625}^{\boxed{1}}$, $3\ 125^{\frac{3}{\boxed{}}}$, $5^{\boxed{3}}$

MIND TEST 1.2j

1. (a) $\dfrac{c^7}{de}$ (b) mn^6 (c) $\dfrac{10x}{3z^2}$

2. (a) $\dfrac{1}{2\ 401}$ (b) 648   (c) 86 400
   (d) $\dfrac{7}{54}$ (e) 81   (f) $\dfrac{125}{8}$

3. 3 456   4. 48

Dynamic Challenge

Test Yourself

1. (a) True   (b) False (25)   (c) False (1)
   (d) False ($32x^{15}$)   (e) True   (f) False $\left(\dfrac{2}{a^4}\right)$
   (g) False $[(\sqrt[5]{32})^2]$ (h) True   (i) False $\left(\dfrac{1}{625\ m}\right)$

2.

$5^{\boxed{4}} \times 5^5$	$5^{3(\boxed{3})}$
$5^{12} \div 5^{\boxed{3}}$	$(\sqrt{25})^{\boxed{9}}$
$\left(\dfrac{1}{5}\right)^{\boxed{-9}}$	$(\sqrt[3]{125})^{\boxed{9}}$
$(5^{\boxed{6}})^{\frac{3}{2}}$	$\dfrac{5^6 \times 5^{\boxed{5}}}{5^2}$
$\dfrac{1}{5^{\boxed{-9}}}$	$\left(\dfrac{1}{5^{\boxed{}}}\right)^3$

3.
2^0 —as→ 1 ; $\dfrac{1}{3^{-4}}$ —as→ 3^4 ; $\left(\dfrac{3}{5}\right)^{-2}$ —as→ $\left(\dfrac{5}{3}\right)^2$; $7^2 \times 5^{-3}$ —as→ $\dfrac{7^2}{5^3}$; $(5^{-1} \times \sqrt{25})^3$ —as→ 1

Skills Enhancement

1. (a) $\dfrac{n^7}{m}$ (b) $\dfrac{x^4 y^7}{2}$ (c) xy^2

2. (a) $\dfrac{4}{125}$ (b) $\dfrac{25}{7}$ (c) 1   (d) 2   (e) 7   (f) 1

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3. (a) 3   (b) 0   (c) −8
   (d) −5   (e) 5   (f) 2
   (g) 2   (h) −1   (i) 1

Self Mastery

1. (a) 1 000   (b) 500 000   (c) 50
2. (a) $\dfrac{3}{4}$ (b) $\dfrac{3}{2}$ (c) 15
3. (a) −1, 6   (b) 1, −7   (c) −1, 4
4. (a) $x = \dfrac{1}{6}, y = 2$ (b) $x = 1, y = -\dfrac{2}{3}$
5. 12°C   6. RM27 130   7. RM61 462.77

CHAPTER 2   Standard Form

MIND TEST 2.1a

1. (a) 2 s.f.   (b) 5 s.f.   (c) 5 s.f.   (d) 4 s.f.
   (e) 2 s.f.   (f) 5 s.f.   (g) 4 s.f.   (h) 6 s.f.

MIND TEST 2.1b

1. (a) 47 200   47 000   50 000
   (b) 5 260   5 300   5 000
   (c) 306   310   300
   (d) 20.7   21   20
   (e) 8.60   8.6   9
   (f) 5.90   5.9   6
   (g) 0.694   0.69   0.7
   (h) 0.0918   0.092   0.09
   (i) 0.00571   0.0057   0.006
2. (a) 12.02   (b) 2.83   (c) 11.1
   (d) 24   (e) 6.61   (f) 13
   (g) 20   (h) 36.0

MIND TEST 2.2a

1. (a) 3.5×10^1 (b) 4.81×10^2
   (c) 5.075×10^3 (d) 9.725×10^1
   (e) 3.1243×10^3 (f) 9.0×10^{-1}
   (g) 2.3×10^{-1} (h) 3.75×10^{-2}
2. (a) 2.5   (b) 37.5   (c) 423
   (d) 5 070   (e) 91 000   (f) 0.62
   (g) 0.0729   (h) 0.001034   (i) 0.0008504
3. (a) 1.05×10^6 metres   (b) 2.16×10^{11} bytes
   (c) 7.5×10^{11} litres   (d) 9.5×10^{-5} metres
   (e) 1.23×10^{-7} metres   (f) 8.9×10^{-17} metres

MIND TEST 2.2b

1. (a) 5.97×10^4 (b) 3.93×10^6
   (c) 1.021×10^8 (d) 1.574×10^5
   (e) 5.46×10^8 (f) 8.59×10^4
   (g) 5.77×10^4 (h) 1.08×10^{-3}
   (i) 6.09×10^{-5} (j) 9.91×10^{-3}
   (k) 7.68×10^{-4} (l) 8.685×10^{-6}

MIND TEST 2.2c

1. (a) 1.48×10^8 (b) 3.75×10^{-8}
   (c) 2.52×10^8 (d) 2.12×10^3
   (e) 4.5×10^{-3} (f) 6.4×10^3
   (g) 2.95×10^3 (h) 8.6×10^8
2. 3.126×10^3 3. 63   4. 10^3 micrometres

1. $2.02 \times 10^5 \, m^3$
2. (a) $9.17 \times 10^7 \, km$ (b) $4.44 \times 10^9 \, km$ (c) $4.35 \times 10^9 \, km$

Dynamic Challenge

Test Yourself

1. (a) 24 000 (b) 54 300 (c) 9 000 (d) 300 000
   (e) 5 000 (f) 5.00 (g) 0.28 (h) 40
   (i) 420 (j) 10 (k) 1.04 (l) 502
2. (a) 3.48×10^8 (b) 5.75×10^4 (c) 5.11×10^4
   (d) 2.96×10^9 (e) 8.84×10^{-2} (f) 3.31×10^{-4}
   (g) 9.77×10^{-8} (h) 5.43×10^4
3. (a) −2, 0.025, 0.025, 1.35, 1.375
   (b) −3, 0.0034, 5.74, 0.0034, 5.7434
   (c) −3, 0.0042, 1.75, 0.0042, 1.7458
   (d) −3, 0.0043, 3.7, 0.0043, 3.657
4. (a) 1.2×10^4 (b) RM214 **5.** 97 people

Skills Enhancement

1. (a) $5.57 \times 10^2 \, m^2$ (b) RM10 824
2. (a) (i) $70.9 \, kmh^{-1}$ (ii) $47.1 \, kmh^{-1}$ (iii) $68.4 \, kmh^{-1}$

Self Mastery

1. (a) Mercury = $7.48 \times 10^7 \, km^2$
      Neptune = $7.62 \times 10^9 \, km^2$
      Jupiter = $6.14 \times 10^{10} \, km^2$
   (b) $6.133 \times 10^{10} \, km^2$
2. (a) 4.37 g (b) 4.99 g

CHAPTER 3 Consumer Mathematics: Savings and Investments, Credit and Debt

MIND TEST 3.1a

1. ● For a well-planned life in the future
   ● As an additional income
   ● For emergency use
2. ● Open a Fixed Deposit Account
   ● This is because the money will not be used for a given period
   ● Higher interest rates are also offered
3. Cheques are commonly used by businessmen/ businesswomen for payments in large amounts while most people only make daily payments in small amounts.

MIND TEST 3.1b

1. RM610.10 **2.** RM1 159.70 **3.** RM106.17

MIND TEST 3.1c

1. Return on investment is the value of return of the investment.
2. (a) RM2 000
   (b) RM24 000 + RM230 000 = RM254 000
3. RM320

MIND TEST 3.1d

1. The higher the risk, the higher the return.
2. Bank Negara Malaysia guarantees on deposits in the bank.
3. It can be cashed immediately.
4. Real estate's price usually increases but rarely falls.

5. (a) Real estate
   (b) Risk potential = Low
      Return = High
      Liquidity = Low
   (c) Mr Osman's action is wise because our country focuses on the tourism sector. Therefore, it is appropriate to set up the homestay. Besides, the investment in the homestay has low risk.

MIND TEST 3.1e

1. Purchase of shares every month or periodically but not at a lump sum.
2. (a) Investor 2. This is because the purchase of 2 shares on a regular basis allows him to purchase many units of shares and the average cost per unit can be reduced.
   (b) RM1.80. 13 268 units of shares
   (c) ● The average cost per unit share can be reduced
       ● Reduce the risk of loss

MIND TEST 3.1f

1. (a) Mr Rasamanie – Real estates (Low Risk)
      Mr Nik Izwan – Savings (Low Risk)
                    Real estates (Low Risk)
                    Shares (High Risk)
   (b) Mr Nik Izwan. This is because if there is a loss in one of the investments, it can be covered by other investments.
   (c) Economic factor and political factor of the location of the real estate.
2. 23.16%

MIND TEST 3.2a

1. Personal loans are short term loans for consumer use.
2. ● Prepare your budget
   ● Plan your expenses
3. Credit card – He is not require to pay interest if his debts are settled in interest-free period as compared to loan.

Dynamic Challenge

Test Yourself

1. Savings is the balance after making mandatory expenditure from salary.
2. ● High interest rate.
   ● Savings period is subject to a specified time.
3. RM8 640

Skills Enhancement

1. Increase the number of shares purchased and the average cost per unit will be lower than if purchased all at once.
2. Purchase of land lots, houses, factories and so on.
3. (a) Dividend (b) Capital gain (c) Bonus share
4. (a) Lee Chong needs to have the knowledge to assess and select shares while Mokhtar's investment is assisted by a professional company.
   (b) Lee Chong's risk is higher compared to Mokhtar's.
5. RM300 **6.** (a) RM360 (b) 3 000 units (c) 9 000 units
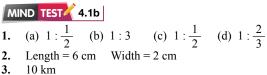7. RM1 000, 3%, 3 years **8.** RM634.12

Self Mastery

1. RM3 750
2. 8.85%
3. RM7 000
4. RM400
5. RM233.33
6. RM52.87
7. (a) Masnah Rasam's view is not recommended because she has to pay interest.
   (b) RM320, 8%
   (c) Cash, because no interest needs to be paid.
8. RM15 000
9. 4%
10. RM900

CHAPTER 4 Scale Drawings

MIND TEST 4.1a

1. Diagram 1, Diagram 2, Diagram 4

MIND TEST 4.1b

1. (a) $1 : \dfrac{1}{2}$ (b) $1 : 3$ (c) $1 : \dfrac{1}{2}$ (d) $1 : \dfrac{2}{3}$
2. Length = 6 cm   Width = 2 cm
3. 10 km
4. 6 cm

MIND TEST 4.1c

2. (b) (i) $1 : \dfrac{1}{2}$ (ii) $1 : 2$

MIND TEST 4.1d

1. 1 944 cm²
2. 34.8 cm
3. 560 m²
4. 20 cm
5. (a) 7 200 m²
   (b) 2 hours 24 minutes

Dynamic Challenge

Test Yourself

1. $1 : \dfrac{1}{5}$
2. (a) I and III
   (b) I = 1 : 2
      III = $1 : \dfrac{1}{2}$
   (c) (i) I = 1.5 cm²
          III = 24 cm²
      (ii) I = 1 : 4
          III = $1 : \dfrac{1}{4}$
      The ratio of area is not proportional to the scale of the scale drawings.
3. (a) 17.0 cm
   (b) 203.5 m²

Skills Enhancement

1. 540 kmh⁻¹
2. 50 cm × 50 cm tile. RM633.20 can be saved.
3. (a) 2 829 m²
   (b) 4 : 13
   (c) 1 971 m²
   (d) RM3 960

Self Mastery

1. (a) 48 m² (b) 8 : 1 (c) 1 440 m³
2. (a) 8 400 m²
   (b) 1 : 500. The most relevant value for scale.
   (c) (i) 60 pieces (ii) RM31 500

CHAPTER 5 Trigonometric Ratios

MIND TEST 5.1a

Angle	Hypotenuse	Opposite side	Adjacent side
∠QPR	PR	QR	PQ
∠PRQ	PR	PQ	QR
∠MNK	KN	KM	MN
∠MKN	KN	MN	KM
∠FEG	EG	FG	EF
∠EGF	EG	EF	FG
∠BAE	AE	BE	AB
∠AEB	AE	AB	BE
∠BCD	CD	BD	BC
∠BDC	CD	BC	BD

MIND TEST 5.1b

1. ΔDEF

$\sin x = \dfrac{EF}{DF}$ $\cos x = \dfrac{DE}{DF}$ $\tan x = \dfrac{EF}{DE}$

$\sin y = \dfrac{DE}{DF}$ $\cos y = \dfrac{EF}{DF}$ $\tan y = \dfrac{DE}{EF}$

ΔKLM

$\sin x = \dfrac{KL}{KM}$ $\cos x = \dfrac{LM}{KM}$ $\tan x = \dfrac{KL}{LM}$

$\sin y = \dfrac{LM}{KM}$ $\cos y = \dfrac{KL}{KM}$ $\tan y = \dfrac{LM}{KL}$

ΔPQR

$\sin x = \dfrac{QS}{PQ}$ $\cos x = \dfrac{PS}{PQ}$ $\tan x = \dfrac{QS}{PS}$

$\sin y = \dfrac{QS}{QR}$ $\cos y = \dfrac{RS}{QR}$ $\tan y = \dfrac{QS}{RS}$

MIND TEST 5.1c

1. Trigonometric ratios of angle x and angle y are the same. This is because all side lengths are reduced by the same rate.
2. (a) (i) $\dfrac{38}{145}$ (ii) $\dfrac{28}{29}$ (iii) $\dfrac{19}{70}$
      (iv) $\dfrac{1}{2}$ (v) $\dfrac{7}{8}$ (vi) $\dfrac{4}{7}$
   (b) No

MIND TEST 5.1d

1. (a) $\sin \theta = \dfrac{15}{39}$ $\cos \theta = \dfrac{12}{13}$ $\tan \theta = \dfrac{15}{36}$
   (b) $\sin \theta = \dfrac{24}{25}$ $\cos \theta = \dfrac{7}{25}$ $\tan \theta = \dfrac{24}{7}$
   (c) $\sin \theta = \dfrac{15}{17}$ $\cos \theta = \dfrac{8}{17}$ $\tan \theta = \dfrac{15}{8}$
   (d) $\sin \theta = \dfrac{5}{13}$ $\cos \theta = \dfrac{12}{13}$ $\tan \theta = \dfrac{5}{12}$
   (e) $\sin \theta = \dfrac{15}{17}$ $\cos \theta = \dfrac{8}{17}$ $\tan \theta = \dfrac{15}{8}$
   (f) $\sin \theta = 0.6$ $\cos \theta = 0.8$ $\tan \theta = 0.75$